

An evaluation of directional analysis techniques for multidirectional, partially reflected waves

Part 1: numerical investigations

Evaluation de techniques d'analyse directionnelle pour des vagues multidirectionnelles partiellement réfléchies

Partie I: investigations numériques

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ABSTRACT

Recent studies of advanced directional analysis techniques have mainly centred on incident wave fields. In the study of coastal structures, however, partially reflective wave fields are commonly present. In the near structure field, phase locked methods can be successfully applied. In the far field, non-phased locked methods are more appropriate. In this paper, the accuracy of two non-phased locked methods of directional analysis, the maximum likelihood method (MLM) and the Bayesian directional method (BDM) have been quantitatively evaluated using numerical simulations for the case of multidirectional waves with partial reflections. It is shown that the results are influenced by the ratio of distance from the reflector (L) to the length of the time series (S) used in the spectral analysis. Both methods are found to be capable of determining the incident and reflective wave fields when $L/S > 0.5$. The BDM provides, in most cases, more accurate estimates of incident significant wave height, average reflection coefficients and directional spreading.

RÉSUMÉ

Les études récentes des techniques avancées d'analyse directionnelle sont essentiellement dédiées aux champs de vagues incidentes. Cependant, dans le cas d'étude d'infrastructures côtières, des champs de vagues partiellement réfléchies sont souvent rencontrés. Dans la zone proche de l'ouvrage, des méthodes assujetties à la phase sont appliquées avec succès. Dans le champ lointain, des méthodes non assujetties à la phase sont plus appropriées. Dans cet article, les performances de deux méthodes d'analyse directionnelle, non assujetties à la phase, la méthode du maximum de vraisemblance (MLM) et la méthode directionnelle Bayésienne (BDM), sont évaluées quantitativement en utilisant des simulations numériques de vagues multidirectionnelles partiellement réfléchies. On montre que résultats dépendent du rapport entre la distance au réflecteur (L) et la longueur de la série temporelle (S) utilisée dans le spectre, et que les deux méthodes sont capables de déterminer les champs de vagues incidentes et réfléchies quand L/S est supérieur à 0.5. La méthode BDM fournit, dans la plupart des cas, des estimations plus précises pour des hauteurs de vagues incidentes significatives, les coefficients de réflexion moyens et l'étalement directionnel.

1 Introduction

The determination of the directional spectrum of real sea waves is a multi-faceted problem, requiring careful consideration of a number of factors. These include the nature of the sea states to be discerned, the type and detail design of the wave measuring system, and the spectral and directional analysis techniques to be employed. Each of these factors can have an interactive effect with the others. The difficulties are compounded by the fact that the sea state is unknown, *a priori*, and thus cannot be used directly to ascertain the accuracy of the derived directional spectra.

It is desirable to utilise a measuring system and associated spectral and directional analysis techniques that are capable of accurately determining the true directional spectrum for multi-directional sea states. Such sea states may encompass simple unimodal incident waves, bimodal incident waves and combinations of such incident waves with reflected waves from either coastal structures or beaches. These reflected waves may or may not be phase-locked to the incident waves at the measurement position.

The emphasis of this paper is to present the finding of a study of the determination of the true directional spectrum for numeri-

cally simulated, multidirectional, partially reflective sea states. The numerical simulations were designed to provide a range of environmental conditions at a particular site, at which an extensive field measurement study had previously been undertaken. Two methods of directional analysis have been used, the Maximum Likelihood Method (MLM), originally introduced by Capon et al (1967) and the Bayesian Directional Method (BDM), introduced by Hashimoto et al (1988). The unique aspect of the paper lies in quantitatively comparing their predictive capacities and showing that the MLM and BDM can be successfully used in the far field zone for partially reflective wave fields.

In section 2 a summary of the relevant background theory is described with special emphasis on spatially distributed arrays, established directional analysis techniques for incident wave fields and how these may be correctly applied to partially reflective wave fields by an appropriate choice of spectral analysis parameters. Section 3 specifies the numerical investigation techniques comprising the generation of synthetic data sets and the spectral and directional analysis techniques. The results are presented in section 4, in which the effects of varying the spec-

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tral analysis techniques are highlighted. These effects are placed within a theoretical framework proposed by Huntley and Davidson (1998), described in section 2.2, which enables an informed choice of spectral analysis technique to be made. The tests using the synthetically generated data also provides the means by which the accuracy of the two methods are quantitatively determined. The results are discriminating allowing clear conclusions to be drawn in section 5.

2 Background

2.1 Directional analysis methods and wave measuring systems for incident waves

A range of directional analysis techniques have been developed and published in the literature. The most prominent of these have most recently been reviewed and classified by Benoit et al (1997). The methods were classified as stochastic methods (in which the phase of individual components is randomly distributed), parametric methods (in which the directional spreading function is assumed to have a particular shape), deterministic methods (which retain the phase information inherent in the data set) and time domain analysis methods (which use time domain variables to estimate the directional spreading function (DSF) rather than frequency domain variables). Based on their conclusions, it is clear that in the case of multidirectional incident sea states the stochastic methods comprising the Maximum Likelihood Method, the Maximum Entropy Method and the Bayesian Directional Method offer superior resolving power when applied to spatially separated multipoint wave gauge arrays. The superior performance of such wave measuring systems, compared to single point systems, was previously demonstrated by Young (1994), who also showed the importance of the array configuration. A laboratory comparison of measurement and analysis techniques carried out by Benoit and Teisson (1994) also arrived at similar conclusions. Chadwick et al (1995) investigated the performance of a star array to measure shoreline directional spectra and found that the MLM produced good estimates, provided that suitable spectral analysis techniques were applied.

2.2 Directional analysis techniques for a reflective wave field

Reflective wave fields caused by coastal structures can be usefully divided into a near structure field and a far field. The near structure field is characterised by the formation of partially standing waves caused by phase locking of incoming and outgoing (reflected) waves. All the stochastic methods were originally developed for a non-phase locked field and therefore fail to give accurate directional spectra estimates in the near field. One method, the modified maximum likelihood method (MMLM), Isobe et al, (1984), which takes in account not only the relative distance between sensors but also their position relative to the reflector, has been successfully used to analyse measurements taken in the near structure environment. Huntley et al (1996) described the sensitivity of the MMLM used for co-located sen-

sors and the problems in applying the method. Ilic et al (1997) showed that this phase sensitive method tends to produce spurious peaks when applied to data measured in the far field. Davidson et al (1998) subsequently demonstrated that this method can produce very accurate estimates in the near field, provided that the effective reflection line is iteratively determined.

For the far field, Huntley and Davidson (1998) postulated that the frequency dependent node/antinode structure implies that for a random wave field, phase-locking between incoming and outgoing waves is linked to the frequency resolution (bandwidth) used in the spectral analysis. If the frequency bandwidth is wide enough to encompass frequencies with both nodes and antinodes of the partially standing waves at the sensor location then in effect the node/antinode structure is smoothed. This would allow the use of non phase-locking methods for data analysis. On the other hand if the frequency bandwidth is small compared to the frequency interval between a standing wave node and antinode, the phase locking effect is significant and a phase-locked method must be used. Huntley and Davidson show further that, since node/antinode frequency interval is determined by the time of wave travel to the reflector and back, L , and the frequency bandwidth in spectral analysis is determined by the length of time series used, S , then the ratio, L/S , can be used to determine whether or not a phase-locked method should be used. In the case of a spatial array, Huntley and Davidson found that the MLM method had full resolution for $L/S > 0.5$. The MMLM method worked well only when $L/S < 0.1$. It can be noted, at this point, that the far field can thus be conveniently considered as that region for which $L/S > 0.5$.

There are two possible ways of increasing the bandwidth. The first one is to decrease the number of data points used in the spectral analysis (e.g. the Fast Fourier Transform segment length), thus to increase the number of segments and frequency bandwidth for a given data series. The second one is to sum the spectral energy over a number of frequencies (e.g. frequency smoothing) for any particular segment length, which also increases the frequency bandwidth. In the present paper, both techniques have been used in analysing synthetically generated multidirectional, partially reflective wave data sets of fixed duration and distance from a reflection line, thus producing L/S ratio's from 0.5 to 5.

2.3 Directional analysis methods used in this paper

Two directional analysis methods have been used in this paper. The first is the Maximum Likelihood Method. This was originally developed by Capon et al (1967) for the analysis of seismic waves with a sensor array. It was later adapted to obtain the directional wave spectrum (see for example Isobe et al (1984)). It is designed to minimise the variance of the difference between the estimate and the true spectrum under the constraint that the amplitude of unidirectional plane waves, with no contamination by noise, is passed without bias.

The estimate that best satisfies this condition is given by Isobe *et al* (1984) as

$$\hat{G}(f, \theta) = \kappa / \left[\sum_m \sum_n \mathbf{P}_{mn}^{-1}(f) H_n^*(f, \theta) H_m(f, \theta) \right] \quad (1)$$

Where, $\hat{G}(f, \theta)$ is the estimate of the directional spreading function (DSF), \mathbf{P}_{mn}^{-1} is an mn element of the inverse cross-spectral matrix $\mathbf{P}^{-1}(f)$ and κ is a proportionality constant, calculated from the condition that the integral of the DSF over 0 to 2π is equal to 1.

This expression also includes the transfer function $H_m(f, \theta)$ between the surface elevation and any other wave property (e.g. pressure, velocity, acceleration) thus enabling directional spreading estimates from measurements of different properties. The symbol * stands for the conjugate operator.

The second method is based on the Bayesian technique used in probability theory. In this method, the estimation of a directional spectrum can be considered as a regression analysis to find the most suitable model from limited data. Therefore, the Bayesian approach can be used to obtain the most reasonable model (DSF) which best approximates the sample (cross-power spectrum) and which also satisfies the nature of physical phenomenon, i.e. a continuous and smooth variation of its value. This can be achieved by maximizing the likelihood of the model with the *a priori* condition that the directional spreading function varies smoothly over direction. The so called hyperparameter (u) is introduced to take in account the balance of the two requirements imposed on the model: to maximize the likelihood of the model and to maintain the smoothness of the model.

Hashimoto *et al* (1987) applied these principles to directional wave analysis. The method is not easy to implement but has powerful capabilities in resolving the DSF shape with no *a priori* assumptions concerning the shape of the DSF. Due to its large number of degrees of freedom, the BDM is able to represent almost all kinds of DSF shapes.

The DSF is expressed as a piecewise - constant function over each segment of the directional range from 0 to $2\pi(k\Delta\theta = 2\pi)$. It is defined by a series of k values x_k (each corresponds to the logarithm of the constant value of the BDM estimate of each segment):

$$\ln[\hat{G}(\theta_k)] = x_k(f), \quad (k = 1, \dots, k) \quad (2)$$

when $\theta_k = (k - 1/2)\Delta\theta$

$$\hat{G}(\theta) = \sum_{k=1}^K \exp(x_k) I_k(\theta) \quad (3)$$

$$\text{with } I_k(\theta) \begin{cases} 1 & \text{if } (k-1)\Delta\theta \leq \theta \leq k\Delta\theta \\ 0 & \text{otherwise} \end{cases}$$

The system of non-linear equations given by the co- and quad spectra is completed by the smoothness condition of the esti-

mated DSF. This is mathematically expressed by the following relationship between three consecutive values of the estimate

$$\sum_{k=1}^K (x_{k+1} - 2x_k + x_{k-1})^2 \rightarrow 0 \quad (4)$$

The most suitable value of the hyperparameter, u , is determined as the one which minimises the Akaike Bayesian Information Criterion (ABIC) given by:

$$ABIC = -2 \ln \int_{-\infty}^{\infty} L(x, \sigma^2) (p x | u^2, \sigma^2) dx \quad (5)$$

where L is the likelihood, p is probability and σ^2 is the variance of a Gaussian distributed random variable.

3 Numerical investigation techniques

3.1 Generation of Synthetic data sets

In order to compare the two methods of prediction, it was considered desirable to generate synthetic data sets which could also be related to suitable field measured data sets. A previous field measurement study (reported in Ilic *et al* (1997)), which included the measurement of the wave field by a spatially distributed array of six pressure transducers located 500m offshore of a system of shore parallel breakwaters, provided such data. The field site, instrument location and wave measurement arrays are illustrated in Figure 1. Corresponding synthetic directional wave data sets were created, therefore, composed of an incident directional spectrum and the corresponding partial reflections from a reflection line set at a distance of 500m. Surface elevation time series at a frequency of 2Hz with a duration of 640 seconds (corresponding to the field measurements) were generated for an array of six positions using the geometry and location of the offshore wave recording system shown in Figure 1.

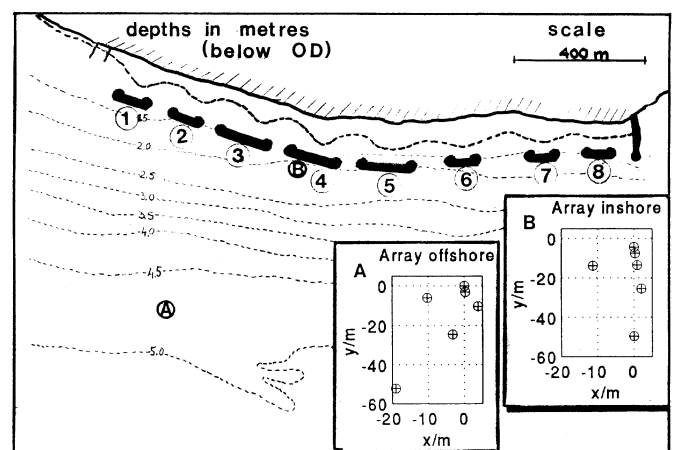


Fig. 1. Field site showing instrumentation location and array configuration.

The surface elevation time series were generated by the technique of white noise filtering in the time domain. This method was introduced by Borgman (1969) for uni-directional waves. It

was extended to multidirectional waves by Christensen (1996) and applied here to include partial reflections.

The generated incident directional spectrum $S(f, \theta)$ given by

$$S(f, \theta) = S(f)G(f, \theta) \quad (6)$$

requires specification of both the frequency spectrum $S(f)$ and the DSF, $G(f, \theta)$. Both JONSWAP and Pierson-Moskowitz type frequency spectra were chosen to cover a wide range of possible wave spectra and a Mitsuyasu type directional spreading function was applied to these frequency spectra. The relevant equations used (taken from Goda (1985)) are as follows:

Frequency spectra

$$S(f) = \frac{aH_s^2 f_p^4}{f^5} e^{-[1.25(\frac{f_p}{f})^4]} \gamma^e \left[\frac{(f-f_p)^2}{2s^2 f_p^2} \right] \quad (7)$$

where

$$a = \frac{0.0624}{\left[0.230 + 0.0336\gamma - \frac{0.185}{1.9 + \gamma} \right]}$$

$$s = 0.07 \text{ for } f \leq f_p$$

$$s = 0.09 \text{ for } f > f_p$$

Choosing $\gamma = 3.3$ results in a JONSWAP type frequency spectrum and choosing $\gamma = 1$ results in a Pierson Moskowitz type frequency spectrum.

Directional spreading function

$$G(f, \theta) = G_0 \cos^2 \left(\frac{\theta}{2} \right) \quad (8)$$

where

$$G_0 = \frac{1}{\pi} 2^{2s-1} \frac{\Gamma^2(s+1)}{\Gamma(2s+1)}$$

In these simulations two values of s were used, $s = 10$ corresponding to wind waves and $s = 25$ corresponding to swell waves with a short decay distance. It should be noted that s was kept constant, rather than varied as a function of frequency.

Two wave periods, 8.0 s and 4.0 s, and three significant wave heights, 0.5 m, 1.0 m and 1.5 m, were selected to represent the measured range of environmental conditions. Three different main wave directions were investigated, normal incidence to the reflector (corresponding to a southerly direction (S) at the field site (refer to figure 1), 10° angle of incidence to the reflector (corresponding towards the south-west direction (SW) at the

field site) and 25° angle of incidence to the reflector (corresponding towards the south-east direction (SE) at the field site). Wave reflection coefficients of 0.2, 0.3, 0.4 and 0.6 were simulated which were frequency independent. Combinations of these different parameter values yielded 45 different wave data sets to be tested (shown in table 1). Additionally, three other data sets were created with a reflection coefficient equal to zero, a JONSWAP type frequency spectrum, a wave height of 1 m and the three different directions.

Table 1. Input parameters for the numerically generated wave data.

Spectrum	Hsi m	Tp s	θ deg	s	Kr
Jonswap	1.0	8.0	10	25	0
Jonswap	1.0	8.0	0	25	0
Jonswap	1.0	8.0	-25	25	0
Jonswap	1.0	8.0	10	25	0.6/0.4/0.3
Jonswap	1.0	8.0	0	25	0.6/0.4/0.3
Jonswap	1.0	8.0	-25	25	0.6/0.4/0.3
Jonswap	1.5	8.0	10	25	0.6/0.4/0.3
Jonswap	1.5	8.0	0	25	0.6/0.4/0.3
Jonswap	1.5	8.0	-25	25	0.6/0.4/0.3
Pierson-Moskowitz	0.5	4.0	10	10	0.4/0.3/0.2
Pierson-Moskowitz	0.5	4.0	0	10	0.4/0.3/0.2
Pierson-Moskowitz	0.5	4.0	-25	10	0.4/0.3/0.2
Pierson-Moskowitz	1.0	4.0	10	10	0.4/0.3/0.2
Pierson-Moskowitz	1.0	4.0	0	10	0.4/0.3/0.2
Pierson-Moskowitz	1.0	4.0	-25	10	0.4/0.3/0.2
Pierson-Moskowitz	1.0	8.0	10	10	0.4/0.3/0.2
Pierson-Moskowitz	1.0	8.0	0	10	0.4/0.3/0.2
Pierson-Moskowitz	1.0	8.0	-25	10	0.4/0.3/0.2

3.2 Spectral Analysis of the numerical simulations

Spectral analysis was performed, as detailed in Chadwick et al (1995), using the Fast Fourier Transform applying a Welch Window, 50% overlapping segments and normalisation of the resulting spectral variance to the time domain variance. Two segment lengths were used, 512 data points at 0.5 second intervals (with additionally frequency smoothing over two frequencies) and 128 data points at 0.5 second intervals. The two cases investigated are summarised in Table 2.

Table 2. Summary of Spectral analysis test cases.

Case No	S (s)	Limits of L/S (Lowest f) (Hz)	Limits of L/S (Highest f) (Hz)	df (Hz)	Degrees of freedom (°)	95% confidence limits
1	256	0.54	1.24	0.0078	16	0.55-2.3
2	64	2.17	4.97	0.0156	30	0.63-1.8

3.3 Directional Analysis of the numerical simulations

The MLM and BDM methods were first applied to the case 1 spectral analysis results specified in table 2 (segment length 512 data points). The results were evaluated by considering the prediction of the main direction, directional spreading, the incident significant wave height and the reflection coefficient. These were only evaluated over frequencies with an energy content > 5% of the maximum energy content.

The main direction and directional spreading were defined in accordance with the latest IAHR guidelines (Frigaard et al (1997)) given by

$$\theta_{M_I} = \frac{\sum_{k=1}^{K/2} \theta_k G(f, \theta_k) \Delta\theta}{\sum_{k=1}^{K/2} G(f, \theta_k)} \quad (9)$$

$$\theta_{M_R} = \frac{\sum_{k=K/2+1}^K \theta_k G(f, \theta_k) \Delta\theta}{\sum_{k=K/2+1}^K G(f, \theta_k)} \quad (10)$$

$$\sigma^2_{\theta_I} = \frac{\sum_{k=1}^{K/2} G(f, \theta_k) (\theta_k - \theta_{M_I})^2 \Delta\theta}{\sum_{k=1}^{K/2} G(f, \theta_k)} \quad (11)$$

$$\sigma^2_{\theta_R} = \frac{\sum_{k=1}^{K/2} G(f, \theta_k) (\theta_k - \theta_{M_R})^2 \Delta\theta}{\sum_{k=1}^{K/2} G(f, \theta_k)} \quad (12)$$

where K is the number of directional segments in 2π radians. The incident significant wave heights obtained by directional analysis were compared to the ‘normalised target’ values. The ‘normalised target’ values were calculated using an expression recommended by Goda and Suzuki (1976) for the significant incident wave height, H_{si} , for irregular waves, defined as

$$H_{si} = \frac{4\bar{\eta}_{rms}}{\sqrt{1 + K_r^2}} \quad (13)$$

where $\bar{\eta}_{rms}$ is the average value of η_{rms} over all six measurement points and K_r is the reflection coefficient.

4 Results of numerical investigations

4.1 Results for main direction with purely incident waves

The main direction predicted by both the MLM and BDM differed only by up to 2° from the target direction over all the data sets.

4.2 Results for directional spreading with purely incident waves

The directional spreading predicted by the MLM was 25% to 41% larger than the target spreading. The largest difference was for the S direction and the smallest for the SE direction. The difference between estimated and target value was 40% to 60% for all directions when the BDM was applied. The differences between the BDM and MLM, expressed as a percentage of the MLM value were from 1.70% to 13%.

4.3 Results for incident significant wave height with purely incident waves

The incident significant wave height (H_{si}) calculated by the BDM was within 1%. The best estimate was obtained for the SE direction and the least accurate for the S direction. The MLM slightly underpredicts H_{si} for the S, gives more accurate predictions for the SW direction and underpredicts for SE. The predicted H_{si} differed by less than 6% of the normalised target values over all data sets and both methods. The direction of wave propagation and array orientation appeared to influence the amount of incident energy detected by both methods.

4.4 Results for reflection coefficient with purely incident waves

Despite using a purely incident wave field, both methods predicted some reflection. The predicted reflection coefficient using the MLM was from 4.1% in the S direction to between 6.9% and 7.1% in the SW and SE directions. The reflection coefficient calculated by the BDM was between 7.3% and 7.6% for the S and SW directions and 8.5% for the SE direction.

4.5 Results for main direction in a partially reflective wave field

Good agreement was found between the main directions predicted using the MLM and BDM. On average, these varied by less than 2%. The influence of the chosen frequency spectra for data simulation on the prediction of main direction had no obvious influence. However, the direction of the incoming waves did influence the prediction of the main direction.

4.6 Results for directional spreading in a partially reflective wave field

The percentage difference between the results obtained by the two methods and the target values are plotted in Figure 2. For a wave period of $T = 8$ sec and directional spread of $s = 25$, both methods produced the largest overprediction, with the MLM providing the better estimate. For wave periods of $T = 8$ sec and $T = 4$ sec and a directional spread of $s = 10$, the smallest differences are observed for the BDM. Overall, both methods overpredict the directional spreading and it is evident that both methods give better estimates for a broad directional spread.

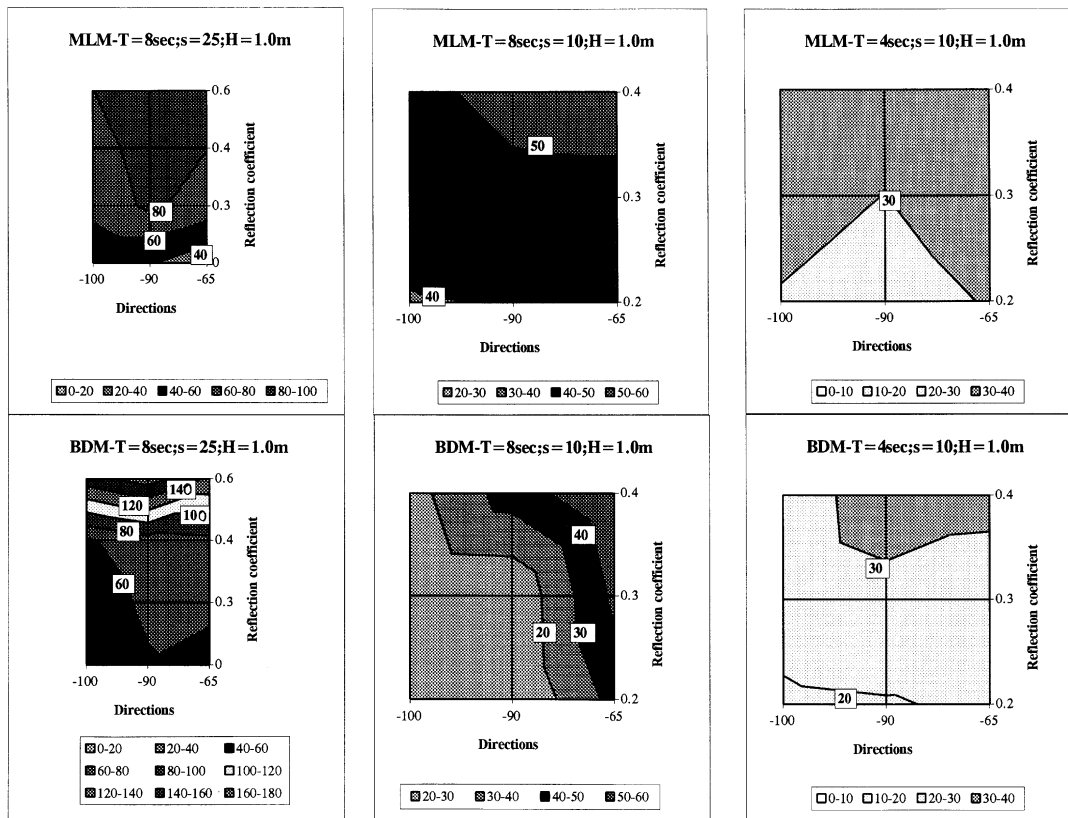


Fig. 2. Percentage differences between target values of directional spreading for the MLM and BDM estimates for a range of wave periods and directional spread, as a function of main direction and reflection coefficient.

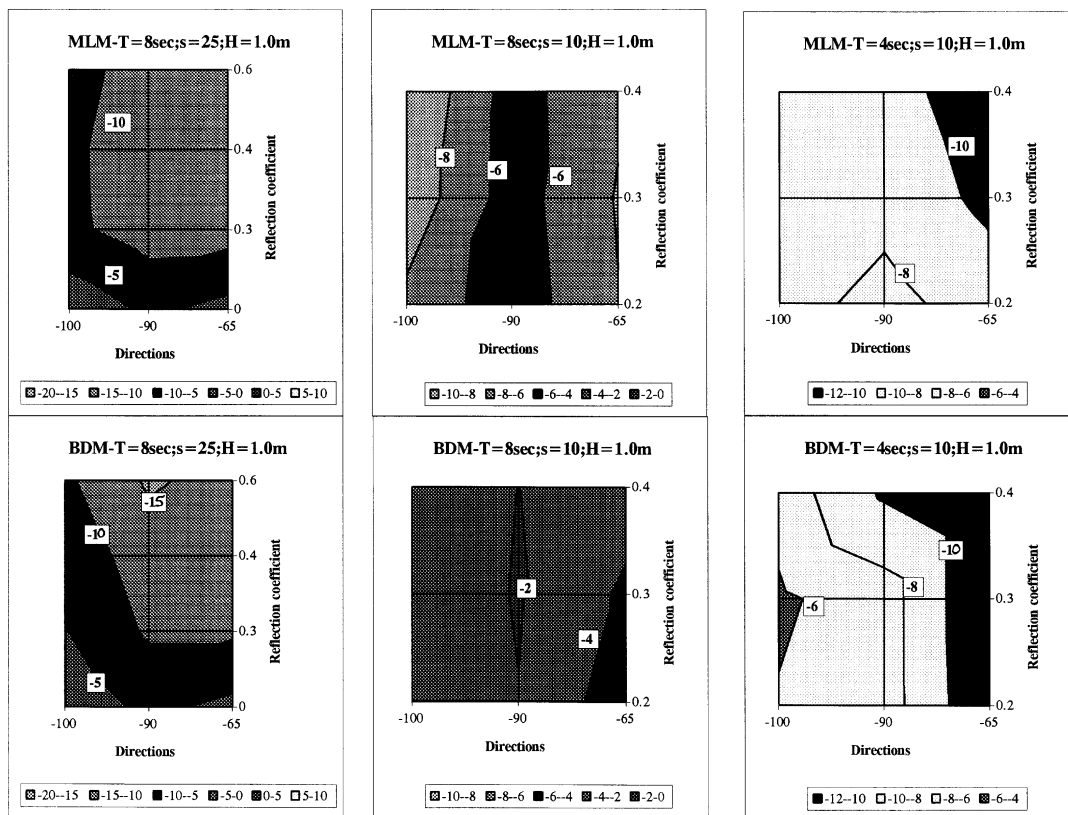


Fig. 4. Percentage differences between target values of incident significant wave height for the MLM and BDM estimates for a range of wave periods and directional spread, as a function of main direction and reflection coefficient.

4.7 Results for incident significant wave height in a partially reflective wave field

Both methods gave quite similar results and it is difficult to distinguish which one gives the best estimates. The influence of the Jonswap and Pierson-Moskowitz type frequency spectra on the results was investigated and no obvious influence of the frequency spectra shape on results was found. However, the influence of the simulated wave direction on the prediction of incident wave height was found to be significant. The best match between predicted and target values was obtained for the SW direction. The predicted values start to deviate from the target values for the S direction and the difference increased for the SE direction. The results are plotted in Figure 3. The MLM produces the lower estimates. Underestimation is in the range of -16% to -2% for both methods. Thus, it was concluded that the array shape and orientation combined with the main wave propagation direction was influencing the results.

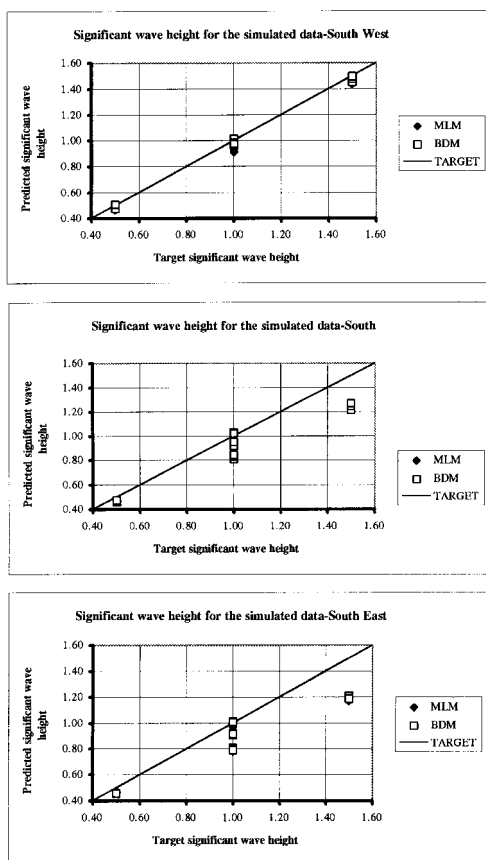


Fig. 3. Predicted incident significant wave height versus normalised target values for the MLM and BDM.

It also became apparent that the reflection coefficient and wave period as well as the directional spread were influencing the prediction of wave height. This is illustrated in Figure 4. For a wave period of $T = 8$ sec and directional spread of $s = 25$, the BDM gave the best estimate from the SW. The MLM estimates became closer to the normalised target value when waves were from the SW and the reflection coefficient was 0.6. The closest estimates were obtained with the BDM method for a wave period of $T = 8$ sec and a directional spread with $s = 10$. Also,

the largest difference between the BDM and MLM was observed for this case (up to 8%). For a wave period of $T = 4$ sec and directional spread with $s = 10$, the percentage difference between both methods' estimates and normalised values increase. Both methods' estimates were more influenced by direction than by reflection and underpredict the H_{si} values. More accurate wave heights were predicted when the reflection was smaller.

4.8 Results for reflection coefficient in a partially reflective wave field

The MLM generally overpredicted the reflection coefficient values. The difference between predicted and target values was between -18% and 120%. The biggest discrepancies occurred in analysing data sets with the lowest reflection coefficient. The influence of spectral energy density shape on prediction of reflection coefficient was investigated and found not to be significant. Figure 5 shows the influence of the main direction of incident wave approach on the relation between predicted and target reflection coefficients. The difference between predicted and target values increases with changing directions from the SW to SE. Generally there is overprediction of the reflection of all three directions using the MLM method. The BDM reflection estimates are closer to the target in the case of the SW direction than in the case of the S and SE direction.

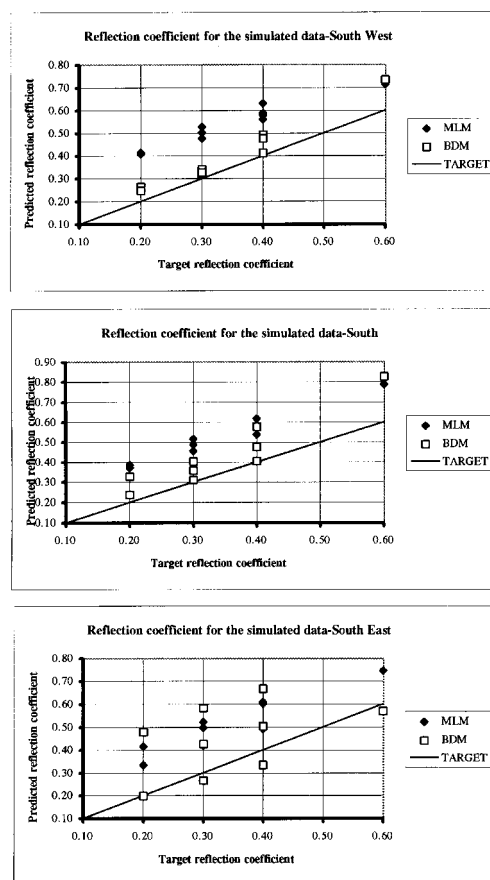


Fig. 5. Predicted reflection coefficient versus normalised target values for the MLM and BDM.

4.9 Reflection as a function of frequency

Even though the reflection coefficient was not pre-set as frequency dependent in the simulations, both methods produced results showing a variation of the reflection coefficient as a function of frequency as shown in figures 6a, b. In the case of the MLM, the values tended to oscillate around a mean value. Conversely the BDM results varied quite markedly with frequency but did not oscillate around a mean value. Despite this, the BDM method produced average reflection coefficient values closer to the simulated coefficient than did the MLM.

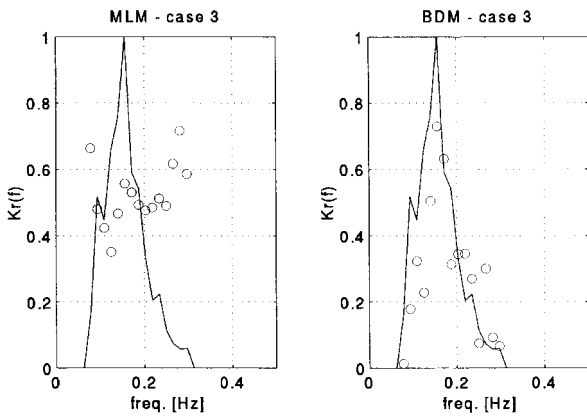


Fig. 6. Frequency dependent reflection coefficient, (a) for MLM and (b) for BDM. Circles represent reflection coefficients. Solid line represents variance as a function of frequency (normalised to the maximum value).

4.10 The influence of segment length on the directional analysis results

As postulated in section 2 the influence of segment length on the directional analysis results was expected to be significant. This was tested by comparing the directional results obtained using the case 1 spectral analysis with those obtained using the case 2 spectral analysis (in which the segment length was reduced from 512 points to 128 points hence increasing the L/S ratio fourfold). The rms values of the difference between estimates and target values obtained by the BDM and the MLM method for incident significant wave height, direction, directional spreading and reflection coefficient have been calculated and are shown in Table 3. In table 4, the method with the lowest error is shown for each set of test parameters. Overall the accuracy of the BDM estimates improve with reducing segment length, the BDM also gives closer estimates of incident wave height, reflection coefficient and directional spreading than does the MLM, but the MLM gives more accurate main directions.

Table 3. Root mean square (rms) values of the difference between estimated and target values.

	rms value for incident wave height		rms value for main direction		rms value for directional spreading		rms value for reflection coefficient	
	MLM	BDM	MLM	BDM	MLM	BDM	MLM	BDM
case 1	0.046	0.044	1.496	2.386	4.964	6.080	0.071	0.042
case 2	0.047	0.039	1.420	1.662	5.281	3.307	0.070	0.036

Table 4. The method with the closest estimates to the target values. B for BDM and M for MLM.

			For Incident significant wave height			For Main direction			For Directional spread			For Reflection coefficient		
			case 1			case 1			case 1			case 1		
			-100	-90	-65	-100	-90	-65	-100	-90	-65	-100	-90	-65
s=25	T=6	K _r	M	M	M	M	M	M	M	M	M	M	M	M
		0	B	B	B	B	B	B	B	B	B	B	B	B
		0.3	B	B	B	B	B	B	B	B	B	B	B	B
		0.4	B	B	B	B	M	B	B	B	B	M	B	B
		0.6	M	M	B	M	B	M	M	M	M	M	M	B
s=10	T=8	K _r	B	M	B	M	M	M	B	B	B	B	B	B
		0.2	B	M	B	M	M	M	B	B	B	B	B	B
		0.3	B	M	B	M	M	M	B	B	B	B	B	B
		0.4	B	M	B	B	B	M	B	M	B	B	B	B
s=10	T=4	K _r	B	B	M	B	B	B	B	B	M	B	B	M
		0.2	B	B	M	B	B	M	B	B	M	B	B	M
		0.3	B	B	M	B	B	M	B	B	M	B	B	M
		0.4	B	M	M	B	B	M	B	B	M	B	M	M
		0.6												
			case 2			case 2			case 2			case 2		
s=25	T=8	K _r	-100	-90	-65	-100	-90	-65	-100	-90	-65	-100	-90	-65
		0.3	B	B	B	B	M	B	B	B	B	B	B	B
		0.4	B	B	B	B	M	M	B	B	B	B	B	B
		0.6		B	M		M	B		B	B		B	B

5 Conclusions

5.1 On the numerical simulations

The directional analysis of the numerical data sets demonstrated that the results obtained were not sensitive to the shape of the frequency spectra. They were, however, sensitive to the value of predetermined reflection coefficient. Larger discrepancies occurred between the values obtained and expected as the input values of reflection coefficient were increased. The results were also influenced by the direction of the incoming waves. The best agreement between predicted and target values were from the SW direction. This was partly due to the array configuration and orientation used in the field. The peak period and directional spread also influenced the results. Usually different combinations of the controlling parameters had a different influence on the BDM than on the MLM.

With regard to the directional analysis method, it was found that both methods can successfully estimate incident and reflected wave fields in the far field where incident and reflected waves are no longer phase locked, provided that a suitable choice of the L/S ratio is possible in practice. Both methods produced very similar results and in some cases the BDM estimates were more accurate. In the other cases the MLM estimates were closer to the target values. However, overall it appears that the BDM provides more accurate results than the MLM, particularly for incident significant wave height, average reflection coefficient and directional spreading.

5.2 On the theoretical framework for phase locked and non phase locked methods

The two methods were applied in the far field for L/S ratios of between 0.5 and 5 (in Huntley and Davidson's zone of full applicability). This analysis has shown that the segment length (S) has an influence on the directional results when the BDM and MLM are applied. The accuracy of the BDM estimates of incident wave height, main direction, directional spreading and average reflection coefficient improve with reducing segment length. The accuracy of the MLM estimates of incident wave height and average reflection coefficient are not strongly

affected by reducing segment length. However, for the MLM, reducing the segment length or increasing the amount of frequency smoothing increases the directional spread and alters the main direction estimates.

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Notations

f	frequency (Hz)
$G(f, \theta_k)$	Directional spreading function
H	transfer function between surface elevation and any other wave property
H_{si}	Incident significant wave height
K_r	Frequency averaged reflection coefficient
P	cross-spectral matrix
s	directional spreading parameter
$S(f)$	spectral energy density (m^2s)
$S(f, \theta)$	directional spectral density (m^2s/rad)
Γ	Gamma function
$\bar{\eta}_{rms}$	Mean root mean square wave height (e.g. averaged over the number of measurement points)
θ_k	Wave direction at interval k
θ_{M_I}	Mean incident wave direction
θ_{M_R}	Mean reflected wave direction
$\sigma_{\theta_I}^2$	Incident directional spreading
$\sigma_{\theta_R}^2$	Reflected directional spreading

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