A simple method for measuring shear stress on rough boundaries

Une méthode simple pour mesurer les contraintes tangentielles sur des parois rugueuses

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ABSTRACT
This technical note presents a simple method for the real time measurement of bed shear stress with a LabView Program for turbulent flow over uniformly rough boundaries, based on the classical logarithmic velocity distribution equation. The method is based on a step-wise linearization of the additive coefficient in the classical logarithmic velocity distribution equation.

RÉSUMÉ
Cette note technique présente une méthode simple pour la mesure en temps réel de la contrainte tangentielle le long du lit à l'aide d'un programme LabView d'écoulement turbulent sur une paroi de rugosité uniforme, basé sur l'équation classique de distribution logarithmique de la vitesse. La méthode est fondée sur une linéarisation par morceaux du terme additif de l'équation de distribution logarithmique de la vitesse.

Introduction

In open channels, turbulent flow over rough boundaries is common and it is often necessary to find the bed shear stress to calculate the velocities and flow rate, possible erosion of the bed as well as the rate of sediment transport. A simple method is to use the Preston tube (Preston 1954), in which the dynamic pressure Δp measured by a total head tube located on the boundary facing the flow, is correlated with the boundary shear stress τ_0 using the law of the wall. For smooth boundaries, the calibration curve provided by Patel (1965) is generally used whereas for uniformly rough boundaries, the calibration curves developed by Hollingshead and Rajaratnam (1980) may be used. In the course of writing a LabView program for real time measurement of bed shear stress on uniformly rough boundaries, it was found necessary to develop a modified procedure and this method is presented herein.

Development of the method

For a Preston tube (which is really a Pitot tube) of external diameter of d placed on an uniformly rough bed with an equivalent roughness height of k_s, facing the flow, neglecting the effects of turbulence and the Pitot displacement effect, the velocity u_0 at the center of the tube, may be assumed to be given by the equation

$$\frac{u_0}{u_*} = 5.75\log[y_0/k_s] + B \tag{1}$$

where y_0 is the distance of the center of the tube from the datum of the rough bed, u_* is the shear velocity, equal to $\sqrt{(\tau_0/\rho)}$;τ_0 is the boundary shear stress; ρ is the mass density of the fluid and B is given by the following set of equations (Nikuradse 1933):

$$B = 5.75\log R_s + 5.5 \quad \text{for } R_s \le 3.5 \tag{2a}$$
$$B = 3.5\log R_s + 6.59 \quad \text{for } 3.5 < R_s \le 7.1 \tag{2b}$$
$$B = 9.58 \quad \text{for } 7.1 < R_s \le 14.1 \tag{2c}$$
$$B = 11.5 - 1.62\log R_s \quad \text{for } 14.1 < R_s \le 70 \tag{2d}$$
$$B = 8.5 \quad \text{for } 70 < R_s \tag{2e}$$

In Eq. 2, $R_s = u_* k_s/\nu$ and the variation of B with R_s is also shown in Fig. 1. Since B is in general a function of the unknown parameter R_s, Eq. 1 may be seen as an implicit equation for calculating the shear velocity u_*.

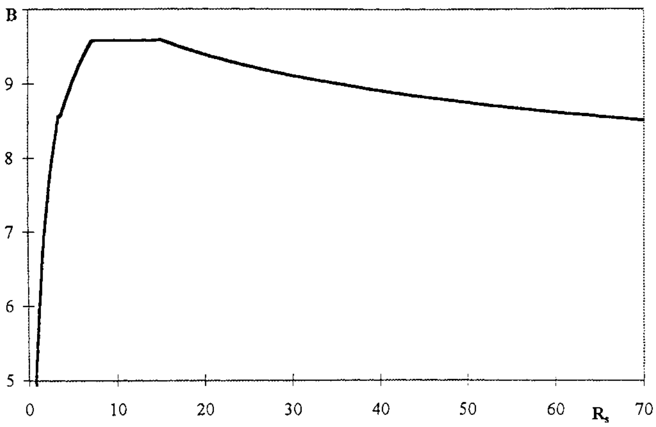

Fig. 1. Variation of B with R_s.

For a given roughness and Pitot (or Preston) tube, the first term on the right hand side of Eq. 1 is a constant, which may be written as A. Multiplying Eq. 1 with R_s,

$$R_0 = AR_s + BR_s \tag{3}$$

where $R_0 = u_0 k_s/\nu$ and R_0 can be calculated for a given fluid, roughness and measured velocity. The first term on the right hand side of Eq. 3 is a linear function of R_s. For $R_s \le 70$, the variation of the second term with R_s is shown in Fig. 2 which is simpler than the variation of B in Fig. 1. When R_s is greater than 70, the second term is also a linear function of R_s, equal to 8.5 R_s. Approximating the nonlinear variation of BR_s by two linear equations (shown as dotted lines in Fig. 2), Eq. 3 is rewritten as

$$R_0 = AR_s + aR_s + b \tag{4}$$

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where *a* and *b* are constants. The constants a and b were found to have the values of 9.94 and –4.70 for R_s in the range of 1.0 to 14.1; 8.30 and 19.50 for R_s in the range of 14.1 to 70 and 8.50 and 0 for R_s 70. Fig. 3 shows the relative error introduced by the linearization of BR_s, which is less than ±2% for R_s in the range of 8 to 70 and less than ±5% for R_s in the range 1to 8.

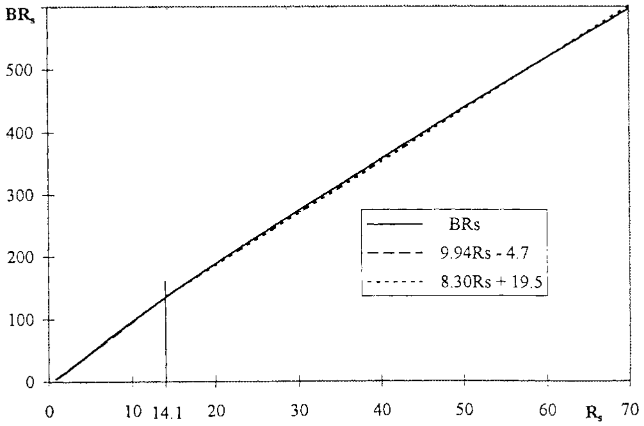

Fig. 2. Variation of BR_s with R_s

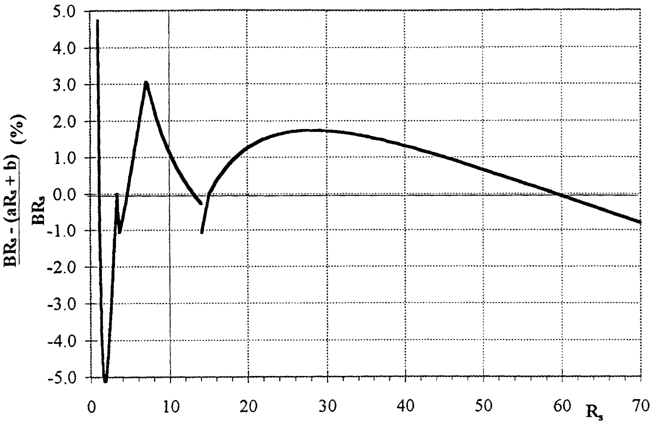

Fig. 3. Variation of the relative error of linearization with R_s

Eq. 4 may be rewritten as

$$R_s = \frac{R_0 - b}{A + a} \tag{5}$$

In order to calculate the shear velocity with Eq. 5, it is effective to start with the (a = 8.30 & b = 19.50) set for R_s in the intermediate range of 14.1 to 70. After calculating R_s from Eq. 5, the proper values of *a* and *b* are obtained to give the final value of u_* from the equation

$$u_* = \frac{u_0 - b\frac{v}{k_s}}{A + a} \tag{6}$$

This technique has been successfully built into a LabView program and has been used to measure the bed shear stress in a project on flow around simple bodies.

Conclusions

A simple method is presented in this note for the real time measurement of bed shear stress for turbulent flow over uniformly rough boundaries, based on the classical logarithmic velocity distribution equation. The technique is based on a step-wise linearization of the additive coefficient in the classical logarithmic velocity distribution equation. The relative error introduced by this approximation has also been assessed.

Appendix I. References

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Appendix II. Notation

The following symbols are used in this note:

A	constant in the velocity distribution equation;
a	coefficient;
B	coefficient in the velocity distribution equation;
b	coefficient;
d	diameter of the Preston tube;
k_s	equivalent sand roughness;
R_0	parameter equal to $u_0 k_s / v$;
R_s	parameter equal to $u_* k_s / v$;
u_0	velocity at the geometric center of the tube of diameter of d;
u_*	shear velocity;
y_0	distance of the geometric center of the tube from the datum;
Δp	dynamic pressure indicated by the tube;
v	kinematic viscosity of the fluid;
ρ	mass density of the fluid;
τ_0	boundary shear stress.