Bed slope effect on the dam break problem

Effet de l'inclinaison du canal sur une rupture de barrage

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ABSTRACT

We consider the dam break problem upon an inclined plane. The flow of a series of Newtonian fluids is generated by the collapse of a dam in a completely transparent channel with variable slope. The stage at given abscissa and the front wave evolution are accurately investigated using ultrasonic and image analysis facilities, respectively. While the Navier Stokes and the continuity equations are properly rendered non dimensional in the viscous regime and solved with the 1D shallow water approximation. A successful comparison is shown between the experimental and the theoretical results.

RÉSUMÉ

Nous étudions ici, le problème de la rupture de barrage sur un canal pentu. L'écoulement d'une série de fluides newtoniens est généré par la rupture d'un barrage dans un canal entièrement transparent et d'inclinaison variable. L'évolution de la hauteur de fluide en une station donnée ainsi que celle du front d'onde sont établies à l'aide de moyens de mesure ultrasonore et d'Analyse d'images, respectivement. Par ailleurs, les équations de Navier Stokes et de continuité sont adimensionnalisées en régime visqueux et résolues avec l'hypothèse des eaux peu profondes. Enfin, les résultats expérimentaux et théoriques sont comparés avec succès.

1 Introduction

The failure of a natural dam can initiate a debris flow [1]. So the knowledge of the flow generated by a dam break is fundamental for the risk prediction in mountainous regions. Indeed, the bed slope is a basic parameter for the collapse of the dam (the hydrostatic pressure is proportional to $\cos(\alpha)$, where α is the inclination of the channel on the horizontal plane) as well as for the development of the flow.

Consider a given quantity of liquid stored in a channel behind an obstructing wall (fig.1) to a depth H at the dam site $x = 0$. At initial time, the dam fails suddenly and completely, then the stage and wave velocity are required to zone the risks downstream. The horizontal dam break problem with an inviscid fluid is governed by the Saint-Venant equations [2,3]. Ritter [4] gives the analytical solution of this set of equations in the form of travelling waves.To take into account the viscosity, Dressler [5] and Whitham [6] introduce in these equations, a friction term depending on the Chezy coefficient. The Saint-Venant equations (hyperbolic system) can also be solved numerically (e.g. Faure and Nahas [7]).

To investigate the inclined dam break problem, Su and Barnes [8] use perturbation series to obtain an approximated solution valid upto the time that the negative wave front reaches the upstream boundary. Hunt [9,10,11,12] gives a kinematic wave solution for steep channels valid after the downstream flood wave has covered a distance of approximately four lengths of the reservoir from the dam site. This asymptotic solution agrees with the numerical computation of the equation of motion performed by Sakkas and Strekoff [13]. Smith [14] gives a similarity solution for a slow viscous steady three dimensional flow down an inclined plane, with application to bottom currents in the ocean. Coussot and Proust [15] and more recently Wilson and Burgess [16] provide a numerical solution of the same flow configuration for a Herschel-Bulkley fluid with application to mudflow. Indeed, the flow of a viscous liquid down an inclined plane is also of fundamental interest with regard to fingering instabilities [17] and of practical interest in coating process (painting...).

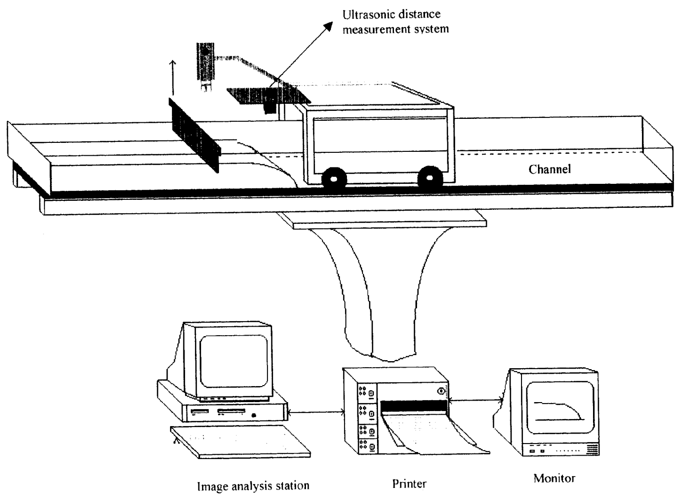

Fig. 1. Experimental device with the channel in the horizontal position.

The aim of the present work is to point out the exact role of the bed slope on the evolution of the front wave and the stage in the flow generated in long domain (as generally occurs for debris flows) by a dam break, with the shallow water approximation, the surface tension being neglected.

The experimental study is presented in the second section. A series of Newtonian fluids are prepared and their severe viscometry is performed. Then their flow generated in a completely transparent channel is generated by a dam failure. The channel is used in it horizontal position and with arbitrary inclination up to 12°. While the flow is investigated using ultrasounds and image analysis techniques. The third section is devoted to the theoretical

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study. The Navier Stokes and the continuity equations are written and properly rendered non dimensional following Piau's scheme [18], and the solution of this system is given for the viscous flow regime. Finally the experimental and the theoretical results are compared in the fourth section.

2 Experimental study

2.1 *Device and procedure*

The experimental set-up is a rectangular one-dimensional *5m* long, 0.30m wide and 0.08m high transparent channel. An obstructing plexiglas plate (same material as the channel itself) located normally to the channel is the dam (fig.2). At negative time, the fluid is poured in the reservoir with height h_0 = 0.055m at the dam site. At initial time, the dam is suddenlylifted. The fluid is released and the flow develops downstream. The flow images are taken in order to investigate the wave front evolution. In the neighbourhood of the dam site, the flow is very rapid. So, an ultra fast camera was used in this work (1000 images/s) able to record the flow initiation for the front wave evolution. These images are shown on a monitor in view to control the quality of the fluid, as will be explained later, and they are simultaneously recorded. Finally, the images are treated using a software pragram (Visilog 4).

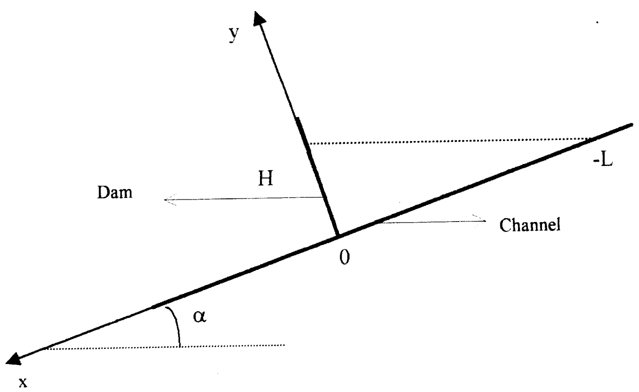

Fig. 2. The boundary conditions.

The time evolution of the stage at given abscissa was investigated using an ultrasonic distance measurement system (Weidmuller LRS 3 Type 262). At a given station, this probe emits a wave normally to the fluid surface. The return wave (after reflection) is in direct relation with the flow depth which is measured with a 0.1mm accuracy.

The fluids used are glucose syrup-water solutions with variable concentration prepared in the following way. an assigned quantity of water is heated in astove at moderate temperature (70˚C). Then, it is slowly incorporated in the glucose syrup using an appropriate propeller which avoids bubbles and solid particles to form. At the end of this operation, the container is closed and the solution is left at rest during 24h. In fact, the glucose-syrup solutions being very sensitive to even a slight change of temperature due to the evaporation, two precautions are taken. First of all, a severe control of the flow temperature using a digital probe (0.05˚C accuracy), and a fluid viscosity test are continuously performed all along the experiment. Furthermore, the absence of bubbles and solid particles is controlled on lie, using the images of the flow displayed on the monitor.

2.2 *Fuid characterization*

Glucose syrup-water solutions were chosen for the present study. Their behavior is Newtonian in the xeperiment and their density as well as their viscosity can be adjusted. Furthermore, these parameters are given in the litterature up to c = 60%, c denoting the concentration of the glucose [19]. Before characterizing the very viscous solutions used here, it was necessary to test less concentrated solutions in view to recover these known values. Figs. 3 and 4 show respectively the variation of the density and the viscosity for higher concentrations (up to c = 100%), exhibiting our results with those of the litterature [19]. These figures show a linear variation of the density. Meanwhile the increasing of the viscosity becomes exponential, according to the Arrhenius law. The density was measured by weighting a given sample of the solution. While the viscosity was measured with a Carrimed CSL 100 rheometer and the results obtained with a Weissenberg rheometer were quite identical.

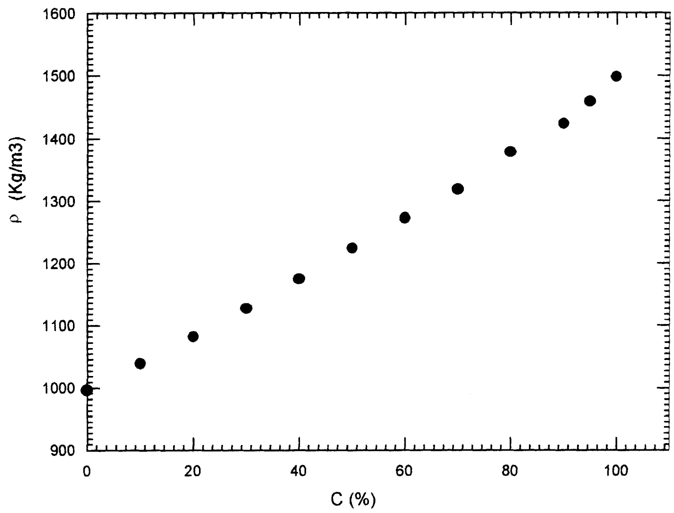

Fig. 3. Density variation the concentration of the glucose syrup in the water.

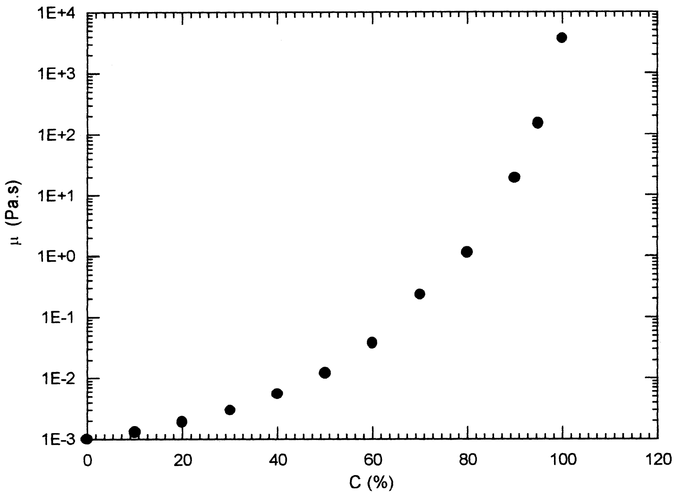

Fig. 4. Dynamical viscosity against the concentration of the glucose syrup in the water.

3 Theory

The flow is described by the Navier-Stokes and the continuity equations, say:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\rho g\sin(\alpha)+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (1)$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}-\rho g\cos(\alpha)+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (2)$$

$$O = -\frac{\partial p}{\partial z} \quad (3)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \quad (4)$$

The coordinate system is presented on fig.1; u and v are the velocity components along the x and y coordinates, respectively; p denotes the pressure field, ρ the fluid density and μ its viscosity.
The flow being dominated by the viscous and the gravity forces, the inertial terms are neglicted and we get after a slight handling:

$$p = \rho g\cos(\alpha)\cdot[h(x,t)-y]+p_0 \quad (5)$$

$$\cos(\alpha)\frac{\partial^2(h^4)}{\partial x^2}-4\sin(\alpha)\frac{\partial(h^3)}{\partial x}-\frac{12\mu}{\rho g}\frac{\partial h}{\partial t} = 0 \quad (6)$$

where $h(x, t)$ denotes the free surface equation and p_0 is the atmospheric pressure.
H being the fluid height at the dam site at negative time and L the reservoir length, define the following set of non dimensional variables:

$$h' = \frac{h}{H};\ x' = \frac{x}{L};\ t' = \frac{\rho g\cos(\alpha)H^3}{12\mu L^2}t \quad (6)$$

eq.(6) becomes

$$\frac{\partial^2(h'^4)}{\partial(x')^2}-4L'\frac{\partial(h'^3)}{\partial x'}-\frac{\partial h'}{\partial t'} = 0 \quad (7)$$

where L' denotes the non dimensional reservoir length and is defined as:

$$L' = \frac{L}{H}\tan(\alpha) \quad (8)$$

Eq.(7) which governs the viscous dam break problem upon an inclined bed is associated with the following initial and boundary conditions (fig.2):

$$\begin{aligned} h'(x',0) &= 0 \quad \text{if} \quad x'>0 \\ &= 1+x' \quad \text{if} \quad x'<0 \end{aligned} \quad (9)$$

$$h'(-1,t') = 0 \quad (10)$$

$$h'(x'_f,t') = 0 \quad (11)$$

where x_f' denotes the non dimensional wave front abscissa.
Hunt [8,9,10] gives an analytical solution of this problem. Far from the front, a kinematic wave approximation is used and an ordinary differential equation is stated for the time evolution of the front:
In the vicinity of the front, Hunt states that the average velocity is independent on the abscissa and a different equation of motion is used [10].
Using the coordinate system as well as the set of non dimensional variables which were introduced here, eq. (7) associated with the initial and the boundary conditions (9-11), were solved following Hunt's method [10].
The kinematic wave approximation produced the following equation of evolution of the front:

$$\begin{aligned} &-72L'^3t'(x_f'+1)+(1+48L'^2t'(L'x_f'+1))^{3/2}- \\ &(1+48L'^2t'(1-L'))^{3/2} = 864L'^5(2-L')t'^2 \end{aligned} \quad (12)$$

which is solved numerically. In the front vicinity, the following equation was obtained for the front position:

$$x_f' = x_s'+\frac{3}{8L'(x_s'+1)}[\ln(4)-1] \quad (13)$$

where

$$x_s' = \left(\frac{3}{4}\right)^{2/3}(12)^{1/3}(t')^{1/3}-1 \quad (14)$$

4 Results

In order to point out the effect of the bed slope on the dam break problem, two series of experiments were performed with the channel in the horizontal position and in an inclined channel.

4.1 *The horizontal channel*

As noticed previously, the time evolution of the wave front and that of the stage at given stations were investigated using video-photographic and ultrasonic facilities, respectively. It is shown in fig.5 where two flow regimes can be identified. In the inertial regime, this evolution follows a Ritter's type law, say

$$x_f' \propto t' \quad (15)$$

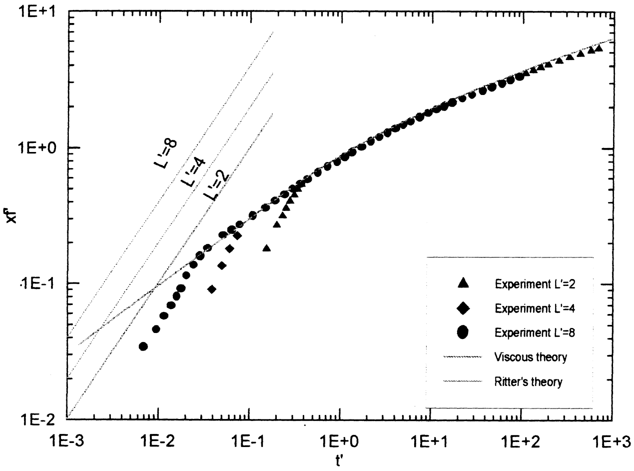

Fig. 5. The front wave evolution for the horizontal channel with: ρ = 1406 kg/m^3; μ = 12 Pa.s; H = 0.55m.

Then, in the initial part of the viscous regime, the law of evolution is

$$x_f' = 0.969t'^{0.5} \qquad \text{if} \qquad t' < 0.1 \tag{16}$$

Finally, this viscous law has the following asymptotic form:

$$x_f' = 1.860t'^{0.2} - 0.902 \qquad \text{if} \qquad t' > 0.1 \tag{17}$$

The typical time variation of the stage at given abscissa is shown in fig.6 (downstream station (•) and upstream station ($_n$). Downstream, where a real danger exists for the people, the fluid depth abruptly increases to a maximum, then decreases and finally tends to an equilibrium value. The trace of these maximum fluid heights is shown in fig. 7; it has the following form:

$$h'_{max} = \frac{0.640}{x' + 1} \tag{18}$$

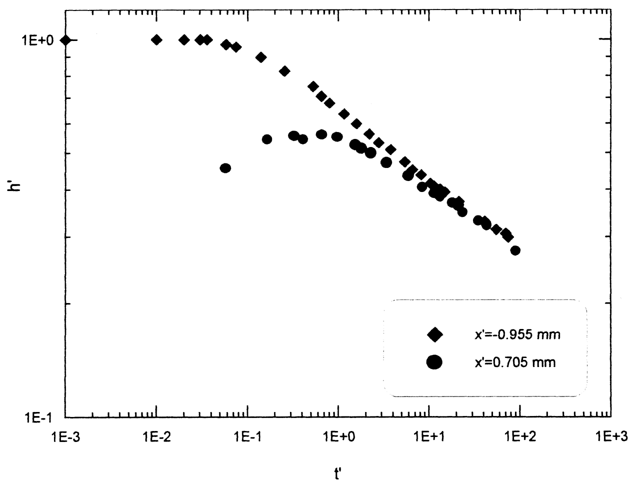

Fig. 6. Typical time height variation at a given downstream station for the horizontal channel with: ρ = 1406 kg/m^3; μ = 12 Pa.s; H = 0.55m.
(•): downstream station (x' = 0.25).
($_n$): upstream station (x' = –0.95) for the horizontal channel.

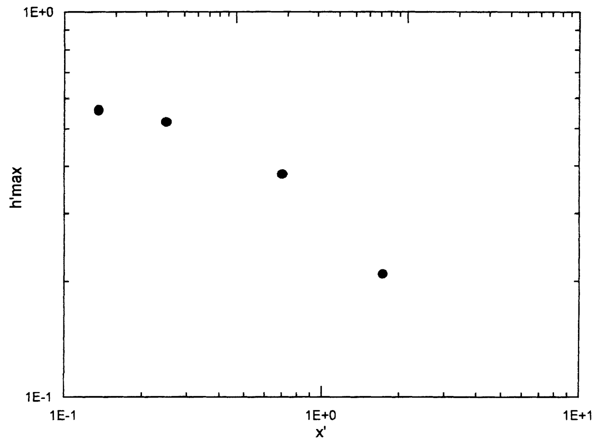

Fig. 7. Trace of the maximum heights for the horizontal channel with: ρ = 1406 kg/m^3; μ = 12 Pa.s; H = 0.55m.

4.2 *The sloping channel*

The front wave evolution is shown in fig. 8. Again, the inertial regime follows a Ritter's type law, say eq. (15). While the initial part of the viscous regime is governed by a similar law as in the horizontal channel, say

$$x_f' = 1.071l'^{0.5} \qquad \text{if} \qquad t' < 1.23 \tag{19}$$

but the asymptotic viscous law is now:

$$x_f' = 2.052t'^{0.31} - 1 \tag{20}$$

The qualitative time variation of the stage at given stations is similar to the horizontal case (see fig.9). The trace of the maximum heights (fig.10) is now governed by the following equation:

$$h'_{max} = \frac{0.553}{(x' + 1)^{0.8}} \tag{21}$$

Finally, note that both series of experiments (horizontal channel and inclined channel) having been performed in the same conditions, the change between the diverse laws which were stated here can be attributed to the bed slope.

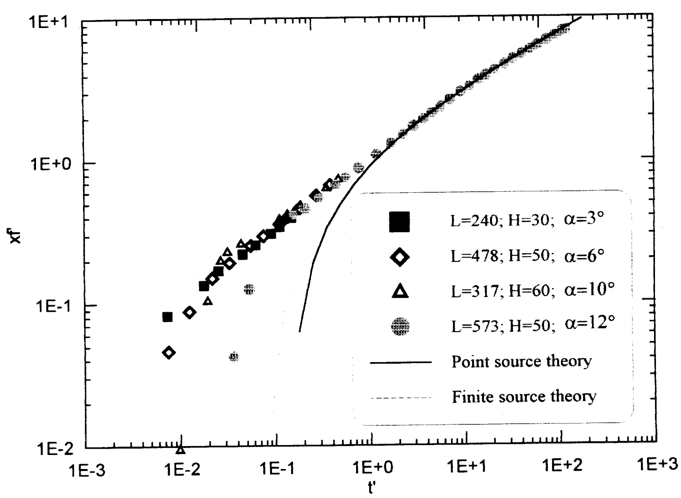

Fig. 8. The front wave evolution for the sloping channel with:

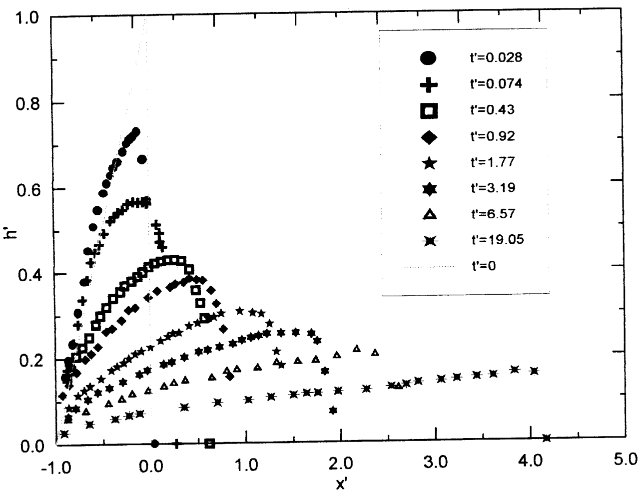

Fig. 9. Time variation of the fluid height for the sloping channel with:

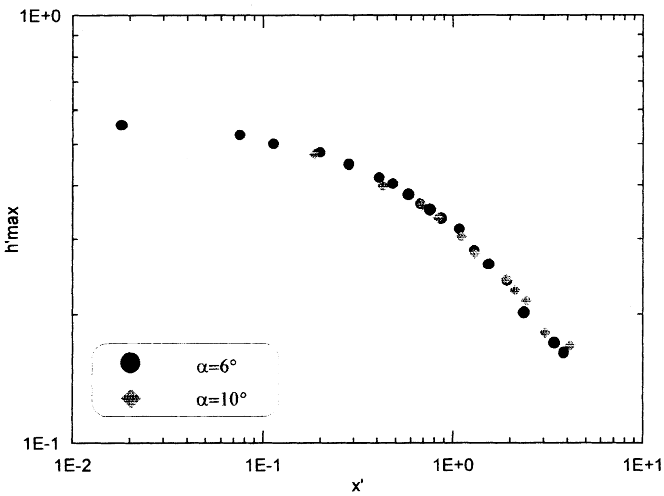

Fig. 10. Trace of the maximum heights for the sloping channel with:

5 Conclusion

The dam break problem upon a sloping channel was investigated both experimentally and theoretically.

In the experimental framework, a completely transparent rectangular channel with a variable slope was made. Then Newtonian fluids say glucose-water solutions were prepared and their rheometry was performed (density and viscosity against concentration of the glucose). The flow of these fluids was then generated by a dam failure with the channel in the horizontal position and in inclined, the slope being varied up to 12°. The wave front evolution was investigated using video-photographic facilities, while the stage evolution at given stations was investigated using ultrasonic facilities.

The theoretical study consisted in a composite method. Far from the front, the flow can be described by a kinematic wave theory. While in the vicinity of the front, a specific equation of motion is written. The comparison between the experimental study and the theoretical study was successful.

Finally it was noticed that as both series of experiments say with the horizontal channel and with the inclined channel were in the same conditions, the change in the previous evolution laws could be attributed to the bed slope.

Notations

c	concentration of glucose-syrup
g	gravity
h	fluid height
h'	non dimensional fluid height
H	fluid height at the dam site at negative time
p	pressure field
p_0	atmospheric pressure
t	time
t'	non dimensional time
x, y, z	cartesian coordinate system with x in downstream direction
x'	non dimensional abscissa along the channel bed
x_f'	non dimensional front abscissa
u, v	velocity components along the x and y coordinates, respectively
α	bed slope
μ	fluid dynamical viscosity
ρ	fluid density

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