Nonuniform sediment transport in alluvial rivers

Transport de sédiments non uniformes en rivière alluviale

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ABSTRACT
A correction factor has been developed in this paper to account for the hiding and exposure mechanism of nonuniform sediment transport. This factor is assumed to be a function of the hidden and exposed probabilities, which are stochastically related to the size and gradation of bed materials. Based on this concept, the formulas to calculate the critical shear stress of incipient motion and the fractional bed-load and suspended load transport rates of nonuniform sediment have been established. These formulas have been tested against a wide range of laboratory and field data and compared with several other existing empirical methods. The predictions by these newly proposed formulas are very good.

RÉSUMÉ
Dans ce papier un facteur de correction a été développé pour tenir compte de l'effet de masquage et d'exposition de sédiments non uniformes en mouvement. Ce facteur est supposé être une fonction des probabilités de masquage et d'exposition, elles-mêmes liées de façon stochastique à la taille des grains et à la distribution granulométrique des matériaux du lit. A partir de ce concept, on a établi les formules pour calculer la tension tangentielle critique de mise en mouvement et les débits solides en charriage et suspension des classes granulométriques de sédiments non uniformes. Ces formules ont été testées sur un large éventail de données de laboratoire et de terrain, et comparées avec plusieurs autres méthodes empiriques. Les prédictions des nouvelles formules proposées sont très bonnes.

1 Introduction

Determining the critical condition for sediment incipient motion and the sediment transport rate is very important in hydraulic engineering. After DuBoys published his research on the bed-load transport rate in 1879 and Shields (1936) proposed the curve for the prediction of the critical bed shear stress of incipient motion, the uniform sediment movement has been extensively investigated and the transport mechanism is reasonably well understood. However, the state-of-art for estimating the nonuniform sediment transport is still inadequate. The pioneering research to fractionally calculate the nonuniform bed-material load transport rate was attributed to Einstein (1950). Afterwards, Egiazaroff (1965), Ashida and Michiue (1971), Hayashi et al. (1980), Qin (1980), Xie and Chen (1982; see Zhang and Xie, 1993), Parker et al. (1982) and Andrews (1983) developed several formulas to determine the incipient motion of nonuniform sediment mixtures. Ashida and Michiue (1971), Parker et al. (1982), Proffit and Sutherland (1983), Misri et al. (1984), Samaga et al. (1986a), Mittal et al. (1990), Bridge and Bennett (1992), Patel and Ranga Raju (1996), and Fang and Yu (1998) proposed several methods for calculating the fractional transport rate of nonuniform bed-load. Hsu and Holly (1992) also proposed a method to predict the gradation of nonuniform bed-load by considering the probability and availability of moving sediment. Samaga et al. (1986b) and Karim (1998) established empirical functions for estimating the fractional transport rates of suspended load and bed-material load.

In the processes of nonuniform sediment movement, the coarse particles on the bed are easier to be entrained than the uniform sediment of equivalent sizes, because they have higher chance of exposure to the flow. The situation is reversed for the fine particles on the bed due to the fact that they are more likely sheltered by coarse particles. Therefore, it is needed to consider the hiding and exposure effect in the modeling of nonuniform sediment transport. Until now, most of the studies on the nonuniform sediment transport are based on introducing some kind of correction factors to account for this hiding and exposure effect and use these factors to modify the existing formulas of uniform sediment transport. In the methods for determining the incipient motion of nonuniform sediment developed by Egiazaroff (1965), Ashida and Michiue (1971), Hayashi et al. (1980), Parker et al. (1982) and Andrews (1983), as well as in the methods for the fractional sediment transport rate developed by Proffit and Sutherland (1983), Bridge and Bennett (1992), Fang and Yu (1998) and Karim (1998), the correction factors were related to bed-material size by

$$\eta_i = f_1\left(\frac{d_i}{d_m}; \text{ or } \frac{d_i}{d_{50}}\right) \tag{1}$$

where d_i is the diameter of the ith fraction of sediment; d_m and d_{50} are the arithmetic mean and 50% sieve diameters of bed materials. Einstein (1950) proposed a comprehensive hiding factor in his bed-load function to account for the interaction effects between coarse and fine particles. After having pointed out the inaccuracy of Einstain's method in the case of discontinuous gradation, Misri et al. (1984) assumed that the motion of fine particles is dominated by the lift force while the motion of coarse particles is by the drag force, and proposed a semi-theoretical hiding-exposure correction factor. This correction factor was revised subsequently by Samaga et al. (1986a), Mittal et al. (1990) and Patel and Range Raju (1996), and in general, it can be expressed as

$$\eta_i = f_2\left(M, \frac{\tau'_b}{\tau_c}, \tau_{*i}\right) \tag{2}$$

Revision received July 20, 2000. Open for discussion till June 30, 2001.

where M is the Kramer's uniformity coefficient; τ'_b is the grain shear stress; τ_c is the critical shear stress for the arithmetic mean size d_m; $\tau_{*i} = \tau'_b/[(\gamma_s - \gamma)d_i]$; and γ_s and γ are the specific weights of sediment and fluid, respectively.

However, the correction factors given in Eq. (1) only involve the non-dimensional grain size d_i/d_m or d_i/d_{50} and can not effectively account for the influence of bed-material gradation. The correction factor proposed in Eq. (2) introduces the Kramer's uniformity coefficient, but it is more complex and thus less attractive in applications. In this study, the hiding and exposure correction factor is related to the bed-material gradation and the hidden and exposed probabilities. Based on this correction factor, the formulas for the incipient motion and the fractional transport rates of nonuniform bed-load and suspended load have been established.

2 New development on the hiding and exposure factor

The drag and lift forces acting on a particle staying on the bed depend on how it is situated and surrounded by others. If there is no other particle on its upstream side, it is exposed completely to the flow and has maximum upwinding area and exposure height; otherwise, its upwinding area and exposure height are reduced. As shown in Fig. 1, we assume that sediment particles are spheres with various diameters and define the exposure height Δ_i for a particle with size d_i as the elevation difference between the apexes of this particle and its upstream particle. If $\Delta_i > 0$, the particle d_i is considered to be at exposed state, and if $\Delta_i < 0$, it is at hidden state. Because sediment particles usually distribute on the bed randomly, Δ_i is a random variable. It is assumed to follow a uniform probability distribution f. If the upstream particle is d_j, f can be expressed as

$$f = \begin{cases} 1/(d_i + d_j), & -d_j \le \Delta_i \le d_i \\ 0, & otherwise \end{cases} \tag{3}$$

The probability of particles d_j staying in the front of particles d_i can be assumed to be the percentage of particles d_j in the bed material, p_{bj}. Therefore, the probabilities of particles d_i hidden and exposed by particles d_j can be obtained from Eq. (3) as

$$p_{hi,j} = p_{bj}\frac{d_j}{d_i + d_j} \tag{4}$$

$$p_{ei,j} = p_{bj}\frac{d_i}{d_i + d_j} \tag{5}$$

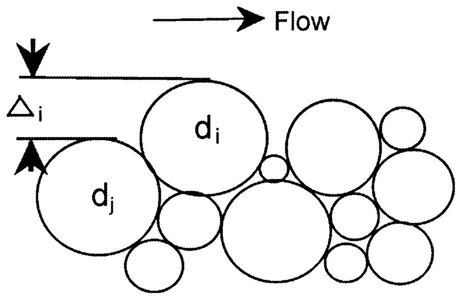

Fig. 1. Definition of exposure height of bed material.

The total hidden and exposed probabilities of particles d_i can be obtained by summing Eqs. (4) and (5) over all fractions, respectively,

$$p_{hi} = \sum_{j=1}^{N} p_{bj}\frac{d_j}{d_i + d_j} \tag{6}$$

$$p_{ei} = \sum_{j=1}^{N} p_{bj}\frac{d_i}{d_i + d_j} \tag{7}$$

where N is the total number of particle size fractions of nonuniform sediment mixtures; p_{hi} and p_{ei} are the total hidden and exposed probabilities of particles d_i. A correlation of $p_{hi} + p_{ei} = 1$ exists between p_{hi} and p_{ei}. By using them, the hiding and exposure factor is defined as

$$\eta_i = \left(\frac{p_{ei}}{p_{hi}}\right)^m \tag{8}$$

where m is an empirical parameter.

In the situation of uniform sediment, $p_{hi} = p_{ei} = 0.5$ and $\eta_i = 1$, which means the hidden and exposed probabilities are equal. In the situation of nonuniform sediment, $p_{ei} \ge p_{hi}$ for the coarse particles, and $p_{ei} \le p_{hi}$ for the fine particles. This can be demonstrated with a simple example. For a sediment mixture with two size fractions $d_1 = 1$ mm, $p_{b1} = 0.4$ and $d_2 = 5$ mm, $p_{b2} = 0.6$, we can get $p_{h1} = 0.7 > p_{e1} = 0.3$, $p_{h2} = 0.3667 < p_{e1} = 0.6333$. It shows that more coarse particles are exposed and more fine ones are hidden.

3 Threshold for incipient motion of nonuniform sediment

Introducing the hiding and exposure factor defined by Eq. (8) to modify the criterion for sediment incipient motion proposed by Shields (1936), we obtain the formula for determining the critical bed shear stress for the incipient motion of nonuniform sediment,

$$\frac{\tau_{ci}}{(\gamma_s - \gamma)d_i} = \theta_c\left(\frac{p_{ei}}{p_{hi}}\right)^m \tag{9}$$

where τ_{ci} is the critical shear stress for particle d_i in nonuniform sediment mixtures; and θ_c can be interpreted as the non-dimensional critical shear stress for the corresponding uniform sediment or the mean size of bed materials. $\theta_c = 0.03$ and $m = -0.6$ are determined by using laboratory and field data in this study.

To verify Eq. (9), it is necessary to know the threshold at which sediment particles start moving. Theoretically, the threshold should be defined as zero bed-load transport rate, but it is not meaningful in practical situation. Many experiments show that even if the flow strength is much weaker than the critical condition proposed by Shields, there are still some sediment particles moving on the bed. Kramer (1935) divided the sediment movement into four stages but his criteria are only qualitative and difficult to apply. Therefore, several low levels of bed-load transport rate were suggested as the quantitative critical condition for incipient motion, for instance, q_b =14cm^3/m/min by U.S. Army Corps of Engineers, Waterways Experiment Station, $q_b/(\rho_s d\omega) = 0.000317$ by Han and He (1984). Yalin (1972) also proposed a quantitative criterion related to the number of moving particles on the bed. For nonuniform sediment, the threshold for incipient motion is more complex because of the hiding and exposure mechanism, etc. Parker et al. (1982) suggested the following threshold for the incipient motion of nonuniform sediment,

$$W_{ri}^* = \frac{q_{bi}(\rho_s/\rho - 1)g}{p_{bi}u_*^3} = 0.002 \qquad (10)$$

where W_{ri}^* is a reference transport parameter; q_{bi} is the volumetric transport rate per unit width for the ith fraction of bed-load; ρ_s and ρ are the specific densities of sediment and fluid; g is the gravitational acceleration; p_{bi} is the gradation of the ith fraction of bed material; and u_* is the bed shear velocity.

We adopt Eq. (10) as the reference transport threshold to determine the critical shear stress. From the collected data shown in Table 1, we can determine the size fractions at incipient motion by using Eq. (10). The corresponding measured bed shear stress is the critical shear stress of these incipient fractions. Fig. 2a shows the comparison of the critical bed shear stresses measured and calculated with Eq. (9). The agreement is very encouraging. The coefficient θ_c is determined as 0.03, rather than in the range of 0.03~0.06 suggested by Shields for uniform sediment.

Table 1. Flow and sediment parameters of bed-load data.

Data Source	Discharge (m^3/s)	Velocity (m/s)	Depth (m)	Energy Slope (10^{-3})	d_i (mm)	q_b ($10^{-3}m^2$/s)
Samaga (1986a)	0.006-0.015	0.49-0.78	0.06-0.11	4.49-6.93	0.073-2.366	0.04-0.22
Kuhnle (1993)	0.01-0.03	0.28-0.81	0.101-0.107	0.47-2.22	0.2-10	0.0000015-0.064
Wilcock (1993)	0.017-0.057	0.26-1.08	0.088-0.12	0.59-16.2	0.21-64	0.00000087-0.22
Liu (1986)	0.0035-0.023	0.14-0.67	0.03-0.083	1.5-4	0.31-30	0.000049-0.00064
Susitna River	799-2800	1.8-2.1	2.4-4.4	1.4-2.4	0.062-128	0.028-0.11
Chulitna River	261-348	1.5-1.8	1.7-1.9	0.64-0.74	0.062-128	0.11-0.23
Black River	20-256	0.44-1.0	0.55-1.9	0.11-0.29	0.062-16	0.0048-0.016
Toutle River	9.3-248	1.3-2.8	0.39-1.5	1.9-5.5	0.062-32	0.11-0.95
Yampa River	26.3-447	0.59-1.3	0.65-3.9	0.40-0.87	0.062-32	0.003-0.054

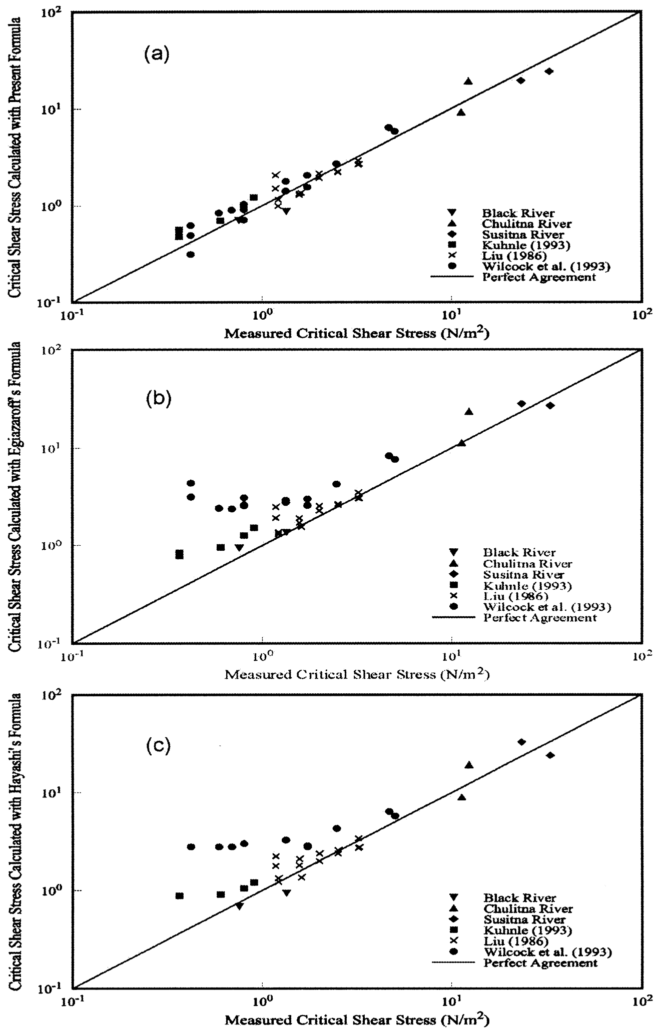

Fig. 2. Measured and calculated critical shear stresses: (a) Newly proposed eq.(9); (b) Egiazaroff's formula; (c) Hayashi et al's formula.

In addition, the present formula (9) is compared with Egiazaroff's (1965) and Hayashi et al's (1980) formulas by using the same data set. Egiazaroff's formula can be written as

$$\frac{\theta_{ci}}{\theta_c} = \left[\frac{\log 19}{\log(19 d_i/d_m)}\right]^2 \qquad (11)$$

and Hayashi et al's as

$$\frac{\theta_{ci}}{\theta_c} = \begin{cases} \left[\dfrac{\log 8}{\log(19 d_i/d_m)}\right]^2 & d_i/d_m \geq 1 \\ d_m/d_i & d_i/d_m < 1 \end{cases} \qquad (12)$$

where $\theta_{ci} = \tau_{ci}/[(\gamma_s - \gamma)d_i]$.

$\theta_c = 0.06$ was used by Egiazaroff (1965), which is too large. Using measurement data Misri et al. (1983) determined $\theta_c = 0.023$~0.0303 in Egiazaroff's and Hayashi et al's formulas. These values are very close to 0.03 used in Eq. (9). For the purpose of comparison in this study, $\theta_c = 0.03$ and the same reference criterion Eq. (10) are used in Eqs. (9), (11) and (12). As shown in Figs. 2a-c, both Hayashi et al's and Egiazaroff's formulas significantly over predict in the range of low bed shear stress. The discrepancy can be attributed to that Egiazaroff's and Hayashi et al's formulas are only related to the grain size, and Hayashi et al's formula uses the same critical shear stress for all the particles finer than d_m. The correction factor used in Eq. (9) is not only related to the grain size but also to the bed-material gradation. Better prediction can be achieved with Eq. (9) even when the bed-material gradation is very wide and highly irregular. Certainly, the comparison of the methods for the critical condition of incipient motion is somehow dependent on the used reference criterion, but the above comparison and the application of Eq. (9) in establishing the nonuniform sediment transport formulas in the next sections show that the newly proposed correction factor has more advantages.

4 Fractional transport rate of nonuniform bed-load

4.1 *Characteristic parameters and relationship*

Meyer-Peter and Mueller (1948), Engelund and Fredsøe (1976), van Rijn (1984) and others related the bed-load transport rate to the excess shear stress $\tau_b - \tau_c$ and other parameters. This type of formulas for the uniform bed-load transport rate or the total transport rate of nonuniform bed-load can be written as

$$\phi_b = f_3\left(\frac{\tau_b}{\tau_c} - 1\right) \tag{13}$$

where ϕ_b is a non-dimensional bed-load transport rate, $q_b/\sqrt{(\gamma_s/\gamma - 1)gd^3}$; q_b is the volumetric bed-load transport rate per unit width; and τ_b is the total bed shear stress or the bed shear stress due to grain roughness.

Eq. (13) is extended to establish the relationship for the fractional transport rate of nonuniform bed-load. The non-dimensional fractional bed-load transport rate ϕ_{bi} is defined as

$$\phi_{bi} = \frac{q_{bi}}{p_{bi}\sqrt{(\gamma_s/\gamma - 1)gd_i^3}} \tag{14}$$

where q_{bi} is the transport rate of the *i*th fraction of bed-load per unit width (m^2/s).

The bed shear stress can be calculated with

$$\tau_b = \gamma R_b J \tag{15}$$

where R_b is the hydraulic radius of channel bed and J is the energy slope.

However, when sand ripples and dunes exist on the bed, it is usually considered that bed-load transport is related only to the grain shear stress, τ_b', which is defined as

$$\tau_b' = \gamma R_b' J \tag{16}$$

where R_b' is the hydraulic radius corresponding to the grain roughness on the bed.

By using the Manning's formula for uniform flow $U = R_b^{2/3}J^{1/2}/n$ and $U = R_b'^{2/3}J^{1/2}/n'$, one can get $R_b' = R_b(n'/n)^{3/2}$. Here U is the average flow velocity, n is the Manning's roughness coefficient for channel bed, and n' is the Manning's roughness corresponding to grain roughness, calculated with $n' = d_{50}^{1/6}/20$ in this study. Therefore, from Eqs. (15) and (16), a method to calculate the grain shear stress can be obtained

$$\tau_b' = \left(\frac{n'}{n}\right)^{3/2}\tau_b \tag{17}$$

Eq. (17) is similar to the method for calculating the grain shear stress adopted by Meyer-Peter and Mueller (1948).

The non-dimensional excess bed shear stress $T_i = \tau_b'/\tau_{ci} - 1$ is used as an independent parameter in the relationship of ϕ_{bi}.

4.2 *Laboratory and field data of bed-load*

Four sets of laboratory data for nonuniform bed-load measured by Samaga et al. (1986a), Liu (1986), Kuhnle (1993) and Wilcock and McArdell (1993) are used to investigate the bed-load transport and the incipient motion in this study. The latter three sets of experiments were for bed-load. In the experiments of Samaga et al. (1986a, b) both bed-load and suspended load existed. The concentration distribution of suspended load along the depth was measured, and the suspended load transport rate was calculated by integrating the concentration distribution and velocity profile from $2d_i$ above the bed to the water surface. At the same time, the total load transport rate was also measured. The bed-load transport rate was obtained by subtracting the suspended load from the total load transport rate. These data sets cover a wide range of flow and sediment conditions, as shown in Table 1.

In addition, some of the field data from five natural rivers (see Williams and Rosgen, 1989) are also used. Because the field measurement of bed-load is very difficult and sometimes inadequate, these data are carefully selected in order to avoid some measurement error. First, the flow and sediment parameters must be measured at the same time. These parameters should include flow discharge, velocity, depth, surface slope, bed-load transport rate, bed-load gradation and bed-material gradation. Secondly, because bed-load may move as strips and at stages, most of the selected bed-load data are those averaged from at least 16 samples across the same cross-section and during a long enough measurement period. Thirdly, the data of bed-material gradation are also averaged from several simultaneous measurement points along the same cross-section to enhance reliability. The ranges of flow and sediment parameters of these field data are included in Table 1.

4.3 *Regressional function*

The collected data of nonuniform bed-load shown in Table 1 are used to establish the relationship $\phi_{bi} \sim T_i$. To calculate τ_b' with Eq. (17), n is determined with $n = R_b^{2/3}J^{1/2}/U$ in this study.

For natural rivers R_b is given as the water depth h because the width-depth ratio of these rivers studied in this paper is large and the influence of banks can be neglected; however, for laboratory experiments the bed and bank shear stresses should be divided and R_b is determined with $R_b = \tau_b/(\gamma J)$. Fig. 3 shows the empirical relationship of $\phi_{bi} \sim T_i$. All the collected laboratory and field data of $T_i > 0$ are included. The data distribute along a straight strip, in which ϕ_{bi} ranges from 10^{-5} to 10^2 and T_i from 10^{-2} to 10^2. By the least square curve fitting, the following formula for the fractional transport rate of nonuniform bed-load is obtained,

$$\phi_{bi} = 0.0053\left[\left(\frac{n'}{n}\right)^{3/2}\frac{\tau_b}{\tau_{ci}} - 1\right]^{2.2} \tag{18}$$

where τ_{ci} is determined with Eq. (9) which has taken into account the hiding and exposure mechanism of nonuniform sediment transport.

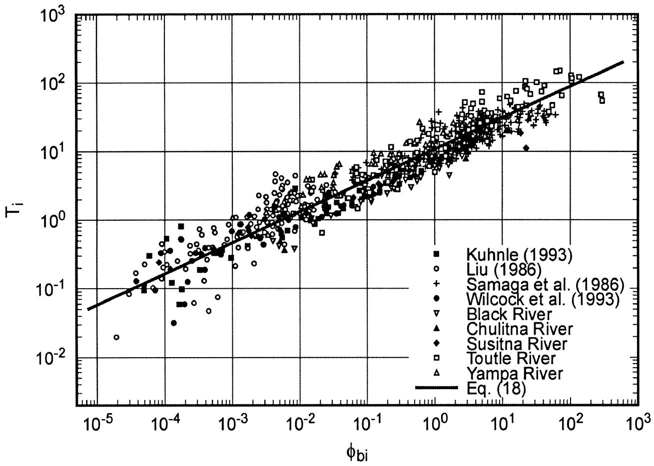

Fig. 3. Relationship for Fractional Transport Rate of Nonuniform Bed-Load.

In Fig. 3, the total number of data points is 752, among which 48.4% points lie within 1/2~2 folds of formula (18), 68.9% in 1/3~3 folds and 81.9% in 1/4~4 folds. The data strip scattering can be attributed to the complexity and stochastic behavior of nonuniform bed-load transport process and the measurement error.

5 Fractional transport rate of nonuniform suspended load

The suspended load transport rate is related to the rate of energy available to the alluvial system, which can be expressed as τU, as explained by Bagnold (1966). Here τ is the shear stress on the entire cross-section, $\tau = \gamma RJ$, and R is the hydraulic radius of channel. In addition, the suspended load transport rate is also influenced by the gravity, which can be accounted for by the settling velocity ω and the critical shear stress τ_c. According to dimensional analysis, the independent parameter $\tau U/\tau_c\omega$ is obtained. Physically, it is more meaningful to replace τ/τ_c with $(\tau - \tau_c)/\tau_c$. Therefore, the fractional transport rate of nonuniform suspended load has relationship,

$$\phi_{si} = f_4\left[\left(\frac{\tau}{\tau_{ci}} - 1\right)\frac{U}{\omega_i}\right] \tag{19}$$

where $\phi_{si} = q_{si}/[\rho_{bi}\sqrt{(\gamma_s/\gamma - 1)gd_i^3}$, and q_{si} is the transport rate of the ith fraction of suspended load per unit width (m^2/s). The laboratory data of nonuniform suspended load measured by Samaga et al. (1986b) and two sets of field data in the Yampa River and the Yellow River are used here to analyze the relationship in Eq. (19). The flow and sediment parameters of these data are listed in Table 2. Fig. 4 shows the relationship of $\phi_{si} \sim (\tau/\tau_{ci} - 1)U/\omega_i$. All the data points scatter in a straight strip, the trend of which can be expressed as

$$\phi_{si} = 0.0000262\left[\left(\frac{\tau}{\tau_{ci}} - 1\right)\frac{U}{\omega_i}\right]^{1.74} \tag{20}$$

where τ_{ci} is determined with Eq. (9). The settling velocity of sediment particles in Eq. (20) is calculated with the Zhang's formula, $\omega_i = \sqrt{(13.95\nu/d_i)^2 + 1.09(\gamma_s/\gamma - 1)gd_i} - 13.95\nu/d_i$ (see Zhang and Xie, 1993). Here ν is the kinematic viscosity.

Table 2. Flow and sediment parameters of suspended load data.

Data Source	Discharge (m^3/s)	Velocity (m/s)	Depth (m)	Energy Slope (10^{-3})	d_i (mm)	Concentration (kg/m^3)
Samaga(1986b)	0.006-0.015	0.49-0.78	0.06-0.11	4.49-6.93	0.073-2.366	0.14-2.62
Yampa River	26.3-447	0.59-1.3	0.65-3.9	0.40-0.87	0.062-1	0.58-2.9
Yellow River	578-5340	0.58-2.38	0.72-4.02	0.18-0.30	0.01-1	7.35-102

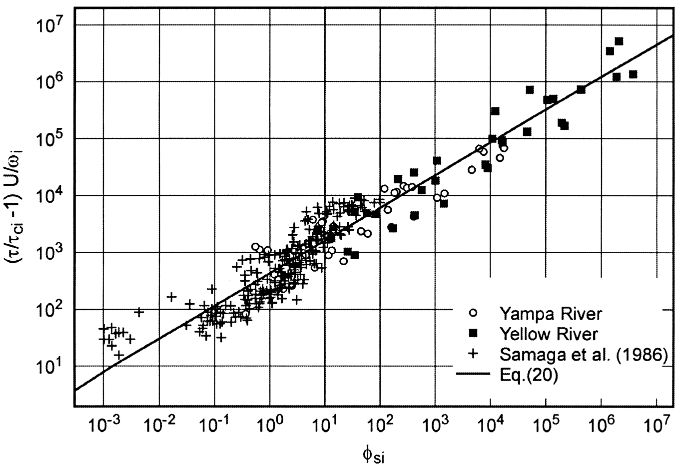

Fig. 4. Relationship for Fractional Transport Rate of Nonuniform Suspended Load.

6 Procedure for computing fractional transport rate of bed-material load

The fractional transport rate of nonuniform bed-material load can be obtained by summing the fractional transport rates of bed-load and suspended load calculated with formulas (18) and (20). The following steps are required for this calculation:

1. Divide the nonuniform sediment mixtures into several fractions with different size ranges and determine d_i, ω_i and p_{bi} for each fraction;
2. Calculate the hidden and exposed probabilities p_{hi} and p_{ei};
3. Determine the critical shear stress τ_{ci} for each size fraction with Eq. (9);
4. Calculate the shear stress τ and the bed shear stress τ_b from the known flow velocity, depth and surface slope. For natural rivers, $\tau_b = \gamma hJ$ can be used, and for experimental situations, τ_b should be obtained by eliminating the bank shear stress;
5. Determine the Manning's roughness coefficient n for channel bed (excluding the influence of banks) and the $n'(= d_{50}^{1/6}/20)$ corresponding to the grain shear stress, and then calculate the grain shear stress τ_b' with Eq. (17);
6. Calculate the non-dimensional excess shear stress $T_i = \tau_b'/\tau_{ci} - 1$, and then the fractional transport rate q_{bi} for nonuniform bed-load with Eq. (18).
7. Calculate the parameter $(\tau/\tau_{ci} - 1)U/\omega_i$, and then the fractional transport rate q_{si} for nonuniform suspended load with Eq. (20).
8. Sum q_{bi} and q_{si} to obtain the fractional transport rate for nonuniform bed-material load.

7 Testing of the proposed transport formulas

Eqs. (18) and (20) are jointly tested against 1859 sets of uniform bed-material load data selected from Brownlie's (1981) data collection by limiting the standard deviation of bed material $\sigma < 1.2$ and the Shields parameter $\theta > 0.055$. These data were observed in several decades by many investigators, which cover the flow discharges of 0.00094~297m^3/s, the flow depths of 0.01~2.56m, the flow velocities of 0.086~2.88m/s, the surface slopes of 0.0000735~0.0367, and the sediment sizes of 0.088~28.7mm. None of them was used to establish Eqs. (18) and (20) in the previous sections. The comparison of the calculated and measured transport rates is shown in Table 3. Similar tests using the same data set are also conducted on three widely used bed-material load transport formulas developed by Engelund and Hanson (1967), Ackers and White (1973), and Yang (1973, 1984). One can see from Table 3 that the newly proposed formulas provide very good results. In addition, Eq. (18) is separately tested against 1345 sets of uniform bed-load data selected from the previous 1859 sets of uniform bed-material load data by limiting the Rouse number $\omega/Ku_* > 2.5$. A comparison is also conducted with four widely adopted bed-load transport formulas of Meyer-Peter and Mueller (1948), Bagnold (1966), Engelund and Fredsøe (1976) and van Rijn (1984). As shown in Table 4, the new formula (18) provides the best results. Van Rijn's formula's results are not very good, because the used data exceed its applicability size range of 0.2~2mm suggested by van Rijn (1984).

Table 3. Comparison of calculated versus measured transport rates of uniform bed-material load.

Error Ranges	Percentages (%) of Calculated Transport Rates in Error Ranges			
	Ackers & White	Yang	Englund & Hanson	Wu et al
0.8≤r≤1.25	37.3	33.4	33.6	40.4
0.667≤r≤1.5	57.9	56.6	55.4	62.7
0.5≤r≤2	82.4	76.6	77.0	81.3

Note: r = calculation / measurement.

Table 4. Comparison of calculated versus measured transport rates of uniform bed-load

Error Ranges	Percentages (%) of Calculated Transport Rates in Error Ranges				
	Van Rijn	Englund & Fredфe	Bagnold	Meyer-Peter & Mueller	Wu et al
0.8≤r≤1.25	14.8	21.4	21.4	21.3	38.7
0.667≤r≤1.5	25.3	37.4	38.9	39.4	59.3
0.5≤r≤2	44.0	54.1	57.2	66.2	80.1

Eqs. (18) and (20) are also jointly tested against the nonuniform sediment data collected by Toffaleti (1968), including the experimental data observed by three groups of investigators: Nomicos, Einstein-Chien and Vanoni-Brooks, and the field data in the Rio Grande River, the Middle Loup River, the Niobrara River and the Mississippi River. In order to avoid the deficiency in the measurement of suspended load close to river bed, the used field data are selected by limiting the lowest measurement point on the depth to be lower than 0.2m (lower than 0.4m in a few of the Mississippi River data). These data cover the flow discharges up to 21,600m^3/s, the flow depths up to 17.5m, and the sediment sizes from 0.062mm to 1mm. The tests are also conducted on the Proffit and Sutherland's (1983) modification of Ackers and White's formula, the modified Zhang's formula (Wu and Li, 1992) as well as Karim's formula (1998). The modified Zhang's formula is only for the fractional transport rate of suspended load, and here it is combined with Eq. (18) to obtain the fractional bed-material load transport rate. Table 5 shows the comparison between the calculated and measured fractional transport rates of nonuniform bed-material load. It is found that the Proffit and Sutherland's method systematically over predicts for these data and provides the worst results. The newly proposed formulas predict the nonuniform bed-material load transport very well.

Table 5. Comparison of calculated versus measured fractional transport rates of nonuniform bed-material load

Data Source and Number	Percentages (%) of Calculated Fractional Transport Rates in Error Ranges											
	0.5 ≤ r ≤ 2				0.333 ≤ r ≤ 3				0.25 ≤ r ≤ 4			
	PS	Zh	Ka	Wu	PS	Zh	Ka	Wu	PS	Zh	Ka	Wu
Flume, 196	10.7	56.6	36.7	61.2	18.4	77.0	52.0	79.6	27.0	87.8	62.2	88.3
Field, 343	2.6	43.2	46.1	56.0	7.0	62.7	70.0	74.1	17.2	76.7	79.6	83.4
Total, 539	5.6	48.1	42.7	57.9	11.1	67.9	63.5	76.1	20.8	80.7	73.3	85.2

Note: PS=Proffit and Sutherland; Zh=modified Zhang; Ka=Karim; Wu=Newly proposed Eqs. (18)&(20)

8 Conclusions

The hiding and exposure effect among the particles of nonuniform bed material is proven important in the prediction of nonuniform sediment transport. A probabilistic model for predicting this effect has been presented in this paper. The hiding and exposure correction factor developed can account for not only the influence of sediment particle size but also that of bed-material gradation. A formula to determine the critical shear stress for the incipient motion of nonuniform sediment has been established based on this correction factor. The comparisons using the laboratory and field data show that the methodology developed has more advantages than Egiazaroff's and Hayashi et al's approaches which only consider the influence of sediment particle size. Furthermore, the formulas for calculating the fractional transport rates of nonuniform bed-load and suspended load have also been established and tested by using a large number of laboratory and field data of both uniform and nonuniform sediment transport. These newly proposed formulas provide better predictions than several existing methods. More research will be conducted in this area and more laboratory and field data will be collected to enhance the accuracy and reliability of these formulas and to broaden their validity in application to a wide range of sediment transport studies.

9 Acknowledgements

This work is a result of the research project supported in part by the Specific Cooperative Agreement No.58-6408-7-035 between the National Center for Computational Hydroscience and Engineering of the University of Mississippi and USDA Agricultural Research Service. Dr. Roger A. Kunhle of the USDA-ARS National Sedimentation Laboratory, Oxford, Mississippi and Mr. Kwonkyu Yu are acknowledged for their consultation and help.

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Notations

The following symbols are used in this paper:

d_i	diameter of the *i*th fraction of sediment
d_m	arithmetic mean diameter of bed materials
d_{50}	50% sieve diameter of bed materials
f	probability distribution function of exposure height
g	gravity acceleration
h	water depth
i	size fraction number
J	energy slope
m	exponent for correction factor
N	total number of particle size fractions
n	Manning's roughness coefficient for channel bed
n'	Manning's coefficient corresponding to grain roughness
p_{bi}	percentage of the *i*th fraction of bed material
$p_{hi,j}, p_{ei,j}$	probabilities of particles d_i hidden and exposed by particles d_j
p_{hi}, p_{ei}	total hidden and exposed probabilities of particles d_i
q_b	bed-load transport rate
q_{bi}, q_{si}	fractional transport rates of bed-load and suspended load
R_b	hydraulic radius of channel bed
R_b'	hydraulic radius corresponding to grain roughness
T_i	non-dimensional excess shear stress
U	average flow velocity
u_*	shear velocity
W^*_{ri}	a reference transport parameter
γ, γ_s	specific weights of water and sediment
η_i	hiding and exposure correction factor for nonuniform sediment
θ_c	non-dimensional critical shear stress of uniform-sediment
θ_{ci}	non-dimensional critical shear stress of the *i*th fraction of sediment
ρ, ρ_s	specific densities of fluid and sediment
τ_b, τ_b'	bed shear stress and grain shear stress
τ_c, τ_{ci}	critical shear stresses for d_m and d_i
ϕ_b	non-dimensional bed-load transport rate
ϕ_{bi}, ϕ_{si}	non-dimensional fractional transport rates of bed-load and suspended load
Δ_i	exposure height of the *i*th fraction of bed material
ω_i	settling velocity of the *i*th fraction of sediment