Numerical simulation of sediment mixture deposition part 2 : a sensitivity analysis

Simulation numérique du dépot sédimentaire en granulométrie étendue 2 : une étude de sensibilité

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ABSTRACT

This paper presents a series of numerical simulations, with the objective of testing the sensitivity to different parameters in the numerical modeling of previously published laboratory experiments. Different options and formulations are analyzed, especially when they concern and influence downstream grading of sediment in rivers. A final discussion deals with the transposition of such investigations to environmental and river engineering studies : refined lab experiments are valuable, despite a direct applicability to real problems which is very limited

RÉSUMÉ

Des travaux de laboratoire et l'analyse qui en a déjà été faite sont ici complétés par une série de simulations numériques. Ces simulations cherchent à mettre en évidence la sensibilité à différents facteurs. Les différentes options et formulations de la littérature sont commentées, en particulier celles qui concernent le tri-granulométrique longitudinal des sédiments. On s'interroge enfin sur la transposition de tels travaux de recherche pour l'étude du comportement des rivières. En dépit de la difficulté qu'il y a d'utiliser directement leurs résultats pour les problèmes réels, l'expérimentation raffinée améliore la compréhension et la formalisation.

1 Introduction

In a companion article (Belleudy, 2000), referred to hereafter as 'the first part' or 'part 1', an analysis was presented of a flume experiment of sediment mixture deposition carried out in St. Anthony Falls Laboratory (SAFL) and described by Seal et al. (1997). This analysis was performed through numerical simulation of the experiment with SEDICOUP software, usually used by the author for engineering applications in rivers (Belleudy, 1992, Belleudy et Schüttrumpf 1994) . The results of the simulation were used as an extension of the existing measurements by SAFL and a discussion was given of the coupling of sediment transport, bed surface material, and exchanges between transport load, bed surface, and underlying material. Relatively simple modeling concepts, as adopted in SEDICOUP software, were able to describe such coupling.

The validity of the demonstration needs to be supported by a discussion of the modeling. This discussion is presented in this paper, based on a sensitivity analysis of modeling choices (formulae and parameters). The analysis focussed on parameters and modeling options related to sediment grading and transport-bed exchanges. Taking account of the modularity of the SEDICOUP modeling system, we performed a series of simulations, testing different modeling options and sensitivity of the system to these options. We will give a discussion of modeling options (those in SEDICOUP but also other ideas) which is based on the application of the software in simulating the SAFL experiments.

The main directions for these modeling experiments are defined from questions which were eluded in the first part :

- What is the behavior of a threshold effect in the modeling of sediment load on the deposition process ?
- Is the description of the bed material in the vertical direction refined enough?
- Will we obtain different deposition patterns when changing mixture properties?
- How does the grain roughness influence the transport load and the deposition?
- What is the real effect of hiding and exposure in grain sorting and deposition in this experiment?

The different simulations are summarized in Appendix 1. During their presentation and discussion, they will be compared to the reference simulation B which was presented in the first part. Finally, we shall present some reflections about transposition and adaptability of the modeling to real river scales.

2 Transport load

The choice was made in part 1 of the calculation of sediment load with the Engelund-Hansen formulation. Such a choice was guided by the characteristics of the sediment and by observed shear conditions. We do not want to discuss the calibration of the formula but rather its character of continuity which can be compared to the threshold effect present in the formula from Meyer-Peter and Müller (MPM).

Some justification of a non-thresholded formula was made when analysing transport conditions and concluding that simultaneous transport as suspended load for the finest particles was possible. A similar conclusion was reached when comparing transport load and Shield's criterion. Simulation C was nevertheless performed with the MPM transport formula.

Threshold effects appear in simulation C as a rapid deposition of the coarsest classes as they enter the system. Figure 1 plots transport load of different size classes at time 84 hr of the simulation and is to be compared with figure 12 of part 1, which is obtained with the EH, non-thresholded, formula (same load at input).

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The result is a continuous rising of the bed floor on top of the wedge (and even the building of a supercritical slope).

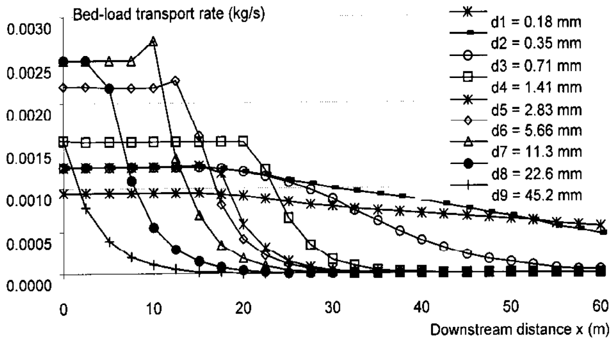

Fig. 1. Profiles of bed load transport rate when using Meyer-Peter & Müller transport predictor (sim. C).

3 Sensitivity to layer thickness

The mixing layer aims at describing sediment at the bed surface exchanging with the transport load. The numerical character of this layer appears when considering the bed material sorting equation.

$$\frac{\partial}{\partial t}[(1-p)\beta_j w_{act} e_m] + \frac{\partial G_j}{\partial x} + \frac{\partial}{\partial t}[(1-p)\beta_j^*(\Gamma - w_{act} e_m)] + S_j = 0 \tag{1}$$

where β_j is volumetric fraction of sediment j at the bed surface and β_j^* is volumetric fraction of sediment j which is considered in exchanges between surface (mixing layer) and subsurface; G_j is the net volumetric sediment discharge of sediment j; Γ is the alluvial bed area with respect to a reference plane ; p is the porosity of bed material ; w_{act} is the width of the 'active bed'; e_m is the mixing layer thickness; and S_j is a source term (exchanges with suspended load, bank aggradation/degradation, lateral input).

In this equation, mixing layer thickness e_m defines the control volume within which sediment continuity is accounted.

The objective of the next run is to test the sensitivity of the modeling to control volume, and the possible bias due to the poor connection between the numerical definition of such a volume and its physical meaning as the 'surface layer' conditioning bed transport and exchanges.

Simulation D has been performed with a different mixing layer thickness $e_m = 0.025$ m (instead of 0.05 m for reference simulation B). At the same time, parameters for strata creation and storage have been reduced, in order to reflect the greater precision in the definition of the deposited material. For numerical reasons (deposition during one time step should not exceed mixing layer thickness), the time step has been reduced proportionally for this sensitivity run.

Compared to the reference calculation, the evolution of the deposition differs only by fractions of millimeters. At equilibrium, elevation of the final deposit and surface mean diameter are strictly identical. The size distribution of the deposited material at the end of the simulation (Figure 2) reflects the differences in strata management, resulting in particular in a more precise definition. Also to be noticed is the difference between elevations 0.15 m and 0.30 m that reflects some differences in sediment deposition on top of the front. The minor differences in upper strata material indicate that the deposition process upstream of the front (i.e. during the 'quasi-equilibrium' stage) is not very sensitive to the change of mixing layer thickness and strata management (such a result must be interpreted with the particular hypotheses of the SEDICOUP calculation in mind).

Because there is no fundamental difference between simulation D and reference simulation B, the next trials will keep the characteristics of the reference simulation for mixing layer and strata parameters. The main reason for such a choice is the reduction of the computation time.

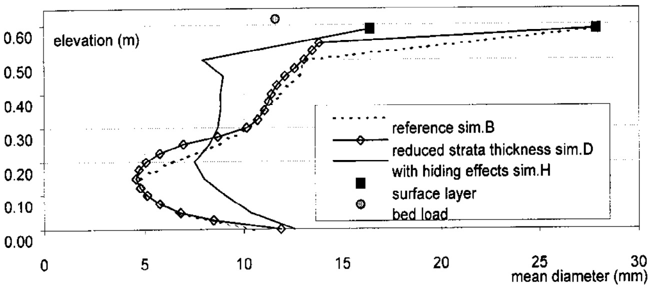

Fig. 2. Mean diameter of sediment deposit (x= 25m).

4 Other grain size distributions

Two different simulations have been conducted with deformation of the sieve curve towards extreme grain sizes, with other conditions and the feed rate being the same as for the reference simulation.

Simulation E was run with an equal mix of the three coarsest classes $d_7 = 11.3$ mm, $d_8 = 22.6$ mm and $d_9 = 45.3$ mm. Figure 3 compares bed profiles at different times with equivalent profiles for the reference simulation. The shear stress that is necessary for bedload transport of larger amounts of coarse sediment requires a steeper friction slope. A consequence of the building of that slope, which requires more material, is a lower front velocity.

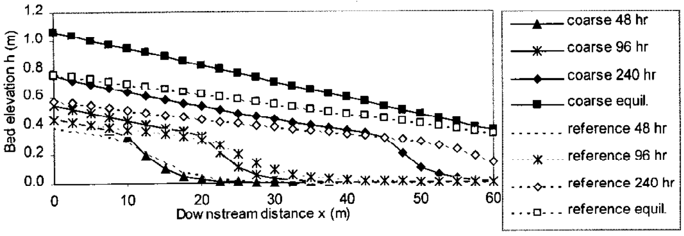

Fig. 3. Bed profiles for simulation E : coarse mixture.

Simulation F was run with an equal mix of three finest classes $d_1 = 0.177$ mm, $d_2 = 0.364$ mm and $d_3 = 0.707$ mm. Deposition (Figure 4) is essentially limited to its initial stages (propagation of front deposit). On top of this initial deposit, a very mild slope is sufficient for transport of the input material. However, the validity of these results must be questioned, considering the fine material in transport; the use of suspended sediment transport procedures (Holly and Rahuel, 1990) would be more appropriate in this case.

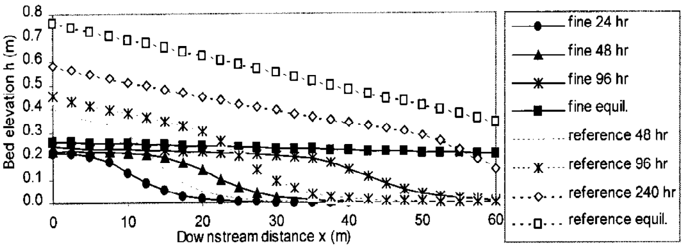

Fig. 4. Bed profiles for simulation F : fine mixture.

5 Roughness

Bedload transport is a direct consequence of shear stress. The more important the roughness is, the higher the shear stress that is exerted by the fluid. The constant and uniform value of bed roughness of the reference simulation (Strickler coefficient $k_{str} = 40$) is replaced in simulation G by a roughness value which is a function of grain size distribution on the bed surface. The classical formulation due to Strickler, with exponent 1/6 of the roughness height (defined as d_{65}) gives values of Strickler roughness coefficient from $k_{str} = 44$ (at the upstream end of the model) to $k_{str} = 60$ (basal sand).

Results of simulation G are not presented here because they are very similar to results from the reference simulation with a constant and uniform roughness. In particular, there is not any noticeable change in bed profile and size distribution when Strickler dependence on roughness height is introduced.

6 Hiding/exposure

Transport capacity g_j for sediment size class j is computed from an adaptation of the formulation by Engelund and Hansen:

$$g_{vj} = 0.1\beta_j \sqrt{g \frac{\rho_s - \rho}{\rho} d_j^3 \frac{\theta_j^{5/2}}{f_{EH}}}$$
$$f_{EH} = \frac{2S_f}{Fr^2} = \frac{2ghS_f}{V^2} \qquad (2)$$

with g the gravitational acceleration; ρ_s, ρ the specific weight of sediment and water, respectively ; d_j the diameter of bed material ; θ_j the dimensionless shear stress ; f_{EH} the friction factor ; S_f the energy slope ; Fr the Froude number ; and h the water depth. The main objection that one could have about equation 2 for the calculation of transport capacity of sediment size class j within the mixture is the lack of any account of hiding and exposure effects. Grain size dispersion of the input sediment, $d_{84}/d_{16} = 4.3$, and surely of the final bed surface mixture, $d_{84}/d_{16} = 45$, are larger than the ratio $d_{84}/d_{16} = 2$ due to Little and Mayer (1976) which is considered as the characteristic value for significant hiding/exposure effects. The reality of such effects was already suggested by our analysis of the mechanism of transport (in part 1).

Our objective is not the selection and calibration of the appropriate formulation but only to point out modifications that will result in the introduction of hiding effects in the simulation. Special attention is paid to front shape and celerity, grain size distribution of the bed surface, final slope and grain size distribution at equilibrium.

Different formulations were proposed for hiding and exposure effects. The original idea, which is recalled by e.g. Sutherland (1992), is the modification of minimum shear stress for the beginning of sediment motion. So-called 'critical shear stress' θ_c is multiplied by a factor lower than one for the finest particles (sheltered on an average by large particles) and larger than one for large particles (because they protrude above mean bed level).

The calculation of transport rate has been extended. In certain cases, it still applies to critical shear stress (when it appears in the transport formulation as in Meyer-Peter and Müller's). In other cases a correction is applied to 'effective' shear stress. In a third case a factor directly modifies transport rate as computed with a single grain size (as for example proposed by Karim and Kennedy 1982). These different types of correction have different meanings and they will behave differently as shear conditions become much larger than at the beginning of motion (say, $\theta > 5\theta_c$).

Ribberink (1987) and Sutherland (1992) among others have compared several of these formulations. SEDICOUP offers to its user those by Karim and Kennedy (1982), by Egiazaroff (1965), and by Day (White and Day, 1982). For the latest , equal mobility for large diameters (at high shear conditions) may be taken in account.

Simulations H and I were run with a formulation according to the principles expressed by Day, which we used for Danube simulations (Söhngen et al., 1992). There is a typical sediment size d_a for which hiding/exposure effects are 'nil'. Sediment finer than d_a will be 'hidden'. Sediment whose characteristic dimension is larger than d_a will be more 'exposed' to flow shear. Exposure coefficient ξ_j is computed for every sediment size class d_j:

$$\xi_j = \left(\frac{d_j}{d_a}\right)^{-\alpha} \qquad (3)$$

Exponent α has a positive value. The literature proposes values between 0.6 and 1.0 (no hiding effects are obtained with $\alpha = 0$). In order to make analysis easier, the simple assumption $d_a = d_m$ has been used. The Engelund and Hansen transport formula, already modified for multiple sediment size classes (equation 2) has been adapted:

$$
\begin{aligned}
g_{\mathrm{vj}} &= \beta_{\mathrm{j}} 0.1 \sqrt{g \frac{\rho_{\mathrm{s}}-\rho}{\rho} d_{\mathrm{j}}^{3}} \frac{(\theta_{\mathrm{j}}/\xi_{\mathrm{j}})^{5/2}}{f_{\mathrm{EH}}} \\
\theta_{\mathrm{j}} &= \frac{hS_{\mathrm{f}}}{\frac{\rho_{\mathrm{s}}-\rho}{\rho} d_{\mathrm{j}}} \qquad (4) \\
\xi_{\mathrm{j}} &= \left(\frac{d_{\mathrm{j}}}{d_{\mathrm{a}}}\right)^{-\alpha}
\end{aligned}
$$

From equation 4, the solid transport rate of size class j depends on volume fraction of sediment j but also on the relative presence of other classes through d_{m}.

Simulations H and I were run with $\alpha = 0.3$ and $\alpha = 0.6$ respectively. Deposition of material into the front zone is concentrated on a shorter front zone as hiding increases. The consequence is a steeper and higher front zone (see Figure 6).

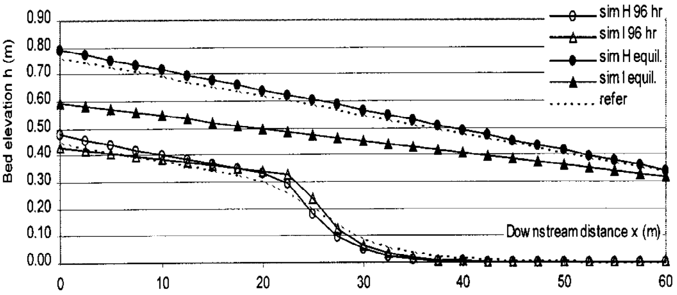

Fig. 6 Modification of transport rate with hiding effect -sim.H: $\alpha = 0.3$ - sim.L:$\alpha = 0.6$

As far as bed elevation is concerned, there is no great difference from the reference simulation when smooth hiding effects are introduced as in simulation H. As the hiding function modifies the transport rate of different size classes, the relative fractions within the mixing layer are adjusted. At equilibrium, the sieve curve (Figure 7) shows a finer mixture than without hiding. In that case, there is only a minor effect on the friction slope.

A very different behavior is observed with simulation I ($\alpha = 0.6$). Bed slope at equilibrium is milder (Figure 6); such a mild slope is in accordance with the relatively fine distribution of sediment on the bed surface at the end of the simulation (Figure 7)

Less dispersion of sediment size is also observed in the whole deposit when hiding effects are introduced (see Figure 2 for simulation H). The longitudinal sorting which appears in the final stage of the deposit has almost completely disappeared with simulation I. A more complex behavior is observed with stronger hiding effects (simulation I), with a convex bed profile and larger grain sizes downstream at the end of front propagation (t = 336 hr), and weak downstream fining and a concave bed profile just after final erosion (t = 384 hr).

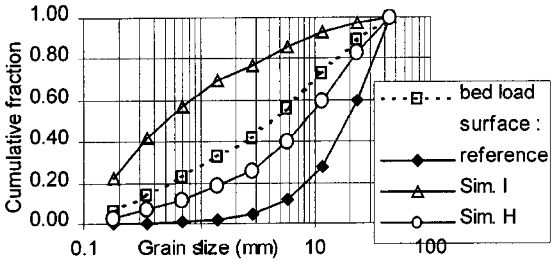

Fig. 7 Sieve curves of bed-load and surface material at equilibrium.

The generic form of the sediment transport rate formula is a power function of shear stress in excess of a critical value. Shina and Parker (1996) have already outlined the importance of the value of the exponent of this power function on bed concavity. A similar conclusion can be drawn for hiding effects. Equation (4), with its 5/2 exponent (insteadt of 3/2 as in most transport formula) multiplies the exponent α of the hiding functions which is then amplified in our calculations. As a result, a much larger fraction of fine sediment is then needed on the bed surface for the same flux; and usual values of α (between 0.6 and 1.0) may be somewhat overestimated.

A strong coupling exists between grain size distribution of the bed surface and transport rate. Calculation by SEDICOUP takes into account the 'mixing layer' grain distribution only. The complexity is even greater with a more complete description (as in that proposed by Di Silvio, 1991) which tends to refine exchanges between bedload and bed surface. For the modeler, strong coupling and complexity will result in extreme difficulty in calibration and validation of his modeling. In real rivers, it appears that the conditions of simulation H, where bed surface grain distribution is not finer than transport load, are more probable.

7 Conclusions : from flume scale to river scale

The question at this point of the paper is the assessment of the differences between SAFL experiments and our simulations on one side, and the behavior and the modeling of real rivers on the other side.

Time !

Time and length are constraints in laboratory experiments and the limited duration is understandable in SAFL runs. Our simulations were pursued after the front reaches the downstream end of the 'channel', which allows 'observation' of dowstream fining that can be compared to downstream fining in rivers (observed without abrasion or hydrological effects, e.g. by Wathen et al., 1995).

Hoey and Fergusson (1994) present numerical simulations at river time and length scales. Their results display development of a strong downstream fining in the case of a concave natural bed profile. Di Silvio and Peviani (1991) also performed simulations with graded sediment, where they reproduced the formation of a concave bed profile from an initial horizontal bed. As outlined by Peviani (1992), such a concave profile is in equilibrium but only on our historical time-scale ; a real equilibrium state, without an abrasion or hydrologic effect, would have a uniform slope and uniform grain distribution, but would require time periods of the order of several centuries.

Initial deposition

The front propagation stage is necessary for building a less artificial initial state in SAFL experiments. At first sight this stage of the run is rather artificial. It can nevertheless be compared to the rapid evolution of the river after sudden input of material due to hydrological or geological hazards in the river or in one of its tributaries.

Loading space lag effects

One could object that introduction of the loading law in our modeling (part 1, equation 3) and the resulting spatial lag could introduce spurious effects. We recognize that space lag has been reported mainly after flume experiments. In real rivers and in the case of bed-load transport, its influence seems to be spatially limited to few decameters at maximum. Space lag is also sometimes equivalent to, and difficult to separate from, time lag.

Let us acknowledge that the loading law equation which is introduced in SEDICOUP modeling is a convenient trick to achieve some smoothing of transport capacity and for modeling of certain types of boundary condition. (i) Smoothing is numerically necessary in order to take account of the variability of 'representative' cross-sections introduced in the model. It describes also in a certain sense the reality of the river because it takes into account partially the 3-D effects that are not modeled (Belleudy and Schüttrumpf, 1994). (ii) Boundary conditions at the upstream end, or internal boundary conditions (downstream tributary or weir in unsteady conditions), result in an 'adaptation' length where transport rate reaches its 'capacity'. Such an adaptation length needs to be calibrated in the modeling. In every case, anyway, the results of the modeling with the loading law will be significant and reliable only if a sufficiently small space interval has been adopted in the model grid for the description of the reach where loading takes place.

The space lag could be of some importance in the case of a predominantly suspended load. In the case when calculation of the suspended load (alone or present together with bed-load transport) cannot be neglected by the simulation, a separate calculation of the different modes of sediment transport is necessary. Holly and Rahuel (1990) describe features available in SEDICOUP. Those features take into account the breakdown of the sediment mixture into several sediment size classes and the interchange of material between suspended sediment, bed-load, and the surface layer.

Mixing layer

The definition of reference material for transport rate calculations is of major importance. In our modeling, this material is wholly contained and defined by the 'mixing layer'. The author agrees with Hoey and Fergusson (1994) that in the case of degradation sediment fluxes towards the surface layer are solely from the subsurface and therefore have the same grain distribution. In the case of degradation we think, however, that definition of reference material is even more important than in the case of aggradation because larger shear stresses are favorable to deeper mixing of surface and subsurface material.

As noticed by Sutherland (1992), the definition of bed surface sediment is of great importance in the evaluation of hiding effects, whatever option has been chosen for its formulation. Preoccupation of scientists concerning division of the active layer into surface and subsurface layers is valuable and will lead to a better understanding of physical behavior. As regards direct application of models for river engineering, we must anyway balance the importance of the question with the uncertainties and schematizations which prevail in engineering studies.

Abrasion

Abrasion at flume scale requires very particular conditions (we can imagine its effect in the case of very friable conglomerates in some industrial processes). Actually, abrasion is essentially the domain of rivers, at certain conditions of sediment properties and if the distance which is considered is sufficient. In natural rivers, the problem in identification and quantification of abrasion will come essentially from its superposition on other sorting phenomena.

The author has never experienced such effects in engineering, essentially because the distances under consideration are often reduced (from 10 to 100 km) and also because their importance seems most often negligible in the case of artificial disturbance of the river. Unless its effects are clearly identified in the river under study, abrasion is an example of those second order effects that could mask the need for validation and calibration of more basic and sensitive processes.

Hydrology and cross section variability

Hydrology is a predominant factor of river morphology and sediment grading, that has been demonstrated essentially by field investigations and already considered by basic research (e.g. Sinha and Parker 1996). This factor is not directly considered by present flume experiments; besides, one can notice a similarity between front propagation and sudden sediment input of relatively fine sediment from an active tributary or landslide.

River morphology and downstream grading is also connected to variability of the cross section in the X direction, which is naturally linked to singularities of geological type and also coupled with hydrological variability. To be considered also are variability of the cross-section in time, especially for important slopes and in the case of braided channels, and variability of grain distribution and shear conditions within the cross section itself (Peviani 1992).

Is black-boxing the only issue for engineers?

The ultimate conclusion is a question concerning applicability of refined modeling concepts to real engineering problems.

Laboratory experiments allow a better identification and comprehension of single processes. Their direct application is the production of 'models', i.e. of some conceptualization and formulas. Such a basic comprehension and consequent refinement of modeling techniques must be integrated by engineers, but with a strong criticism based on a deep expertise.

Those 'models' are easily introduced in software and simulations. But it has been demonstrated that the same effect, e.g. changing front shape, can be obtained with moderate adjustment of the parameters that characterise the main phenomena. Each refinement of the models will introduce a new set of options and parameters whose validation and calibration are more and more problematic, but whose effect on the behaviour of the simulation can be totally misleading.

Is 1-D modeling of real rivers, with its schematisation of real natural complexity of the geometry, worth the refinement of trying to describe every single process?

Our provisional answer is yes, despite the poor validation data that prevail in most engineering projects (at least in the domain of sediment transport). This is because it has the merit of demonstrating the weakness of the simulation and because it leads to more care and expertise in validation and assessment of the confidence that the engineer can have in the simulation results. Finally, because it is a necessary step toward the development of modeling expertise in the domain of river morphology.

Acknowledgements

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Special thanks also to Pr. Peter Ross for his kind collaboration for correction of English expression.

Appendix 1: Characteristics of the different runs

Reference simulation B:

Boundary conditions		
Liquid discharge	Q	0.049 m³/s
Downstream water level	h	0.5 m
solid input	Gs	0.015 kg/s
Sediment density	ρs	2690 kg/m³
Sediment porosity	p	0.33
Model		
Width	b	0.305 m
Length	L	60 m
Space interval	Δx	2.5 m
Strickler roughness coefficientl	k_{STR}	40
Modeling parameters		
Mixing layer thickness	e_m	0.05 m
Thickness of strata	e_{str}	0.05 m
'storage parameter'	a	1.5
Hiding/exposure exponent	α	0 (no effect)

	name	Characteristic feature	
A		presented in part 1 / high sediment input rate	
B	reference	-	
C		thresholded formula for transport load	Meyer-Peter and Müller transport formula
D		Reduced thickness of layers	e_m=0.025m, e_{str}=0.025m, a = 1.1
E		Coarse sediment mixture	
F		Fine sediment mixture	
G		Roughness coefficient, shear stress, are direct functions of d65 of surface layer	
H		Exposure correction factor is effective	$\alpha = 0.3$
I		Exposure correction factor is effective	$\alpha = 0.6$

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