Transport of sediment in large sand-bed rivers

Transport des sédiments dans les grandes rivières à lit de sable

ALBERT MOLINAS, *Associate Professor, Civil Engineering Department, Colorado State University, Fort Collins, CO 80523.*

BAOSHENG WU, *Department of Hydraulic Engineering, Tsinghua University, and Key Laboratory for Water and Sediment Science, Ministry of Education, Beijing 100084, China; formerly Senior Hydraulic Engineer, Hydrau-Tech, Inc., 333 West Drake Rd., Suite 40, Fort Collins, CO 80526.*

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ABSTRACT

A sediment transport equation based on universal stream power is presented for the prediction of bed-material concentrations in large sand-bed rivers. The universal stream power, which is derived from the energy concept, has the advantage of eliminating the energy slope as a parameter. The energy slope, which is in the order of 10^{-5} for large rivers, is a major source of uncertainty in measurements. The analysis shows that relationships derived from flume experiments with shallow flows cannot be universally applied to large rivers with deep flows. Also the use of dimensionless homogeneous parameters in an equation is not sufficient to ensure its applicability to flow conditions where flow depths are several orders of magnitude larger. The comparisons between computed and measured sediment concentrations indicate that the commonly used Engelund and Hansen, Ackers and White, and Yang equations which were developed using mainly flume experiments are not applicable for large rivers with flow depths and Reynolds numbers up to 100 times larger than those found in flumes. The Toffaleti's method which was developed mainly from field data gives reasonable predictions of sediment transport rates for large rivers. Using the proposed equation, the computed sediment transport rates are in much closer agreement with the actual measured values in large and medium rivers.

RÉSUMÉ

On présente une équation de transport de sédiments basée sur la puissance globale du courant pour prédire les concentrations de matériaux dans les grandes rivières à lit sableux. La puissance globale du courant qui est dérivées du concept d'énergie, a l'avantage d'éliminer la pente de la ligne d'énergie comme paramètre. La perte de charge linéaire qui est de l'ordre de 10^{-5} pour les grandes rivières, est une source importante d'incertitude dans les mesures. L'analyse montre que les relations déduites d'expériences sur des écoulements peu profonds en canal ne peuvent pas être universellement appliquées aux grandes rivières avec des écoulements profonds. De même, l'utilisation de paramètres homogènes sans dimension dans une équation n'est pas suffisante pour assurer son applicabilité aux écoulements dont les tirants d'eau sont supérieurs de plusieurs ordres de grandeur. La comparaison entre les concentrations de sédiments calculées et mesurées indique que les équations communément utilisées de Engelund et Hansen, de Ackers et White, et de Yang, qui ont été établies principalement d'après des expériences en canal, ne sont pas applicables aux rivières dont les profondeurs et les nombres de Reynolds sont jusqu'à 100 fois supérieurs à ceux des canaux expérimentaux. La méthode de Toffaleti qui a été développée principalement à partir de données naturelles donne des prédictions raisonnables du transport de sédiments pour les grandes rivières. En utilisant l'équation proposée le transport solide est en accord plus étroit avec les valeurs réellement mesurées dans les rivières grandes et moyennes.

1 Introduction

Sediment transport involves complex interaction between numerous interrelated variables. Theoretical approaches in the study of sediment transport are based on simplified and idealized assumptions. Empirical methods emphasize only certain number of parameters which are considered to be more relevant by their developers. Therefore, the applicability of an equation to estimate the transport rates under field conditions relies not only on the theoretical formulations, but also on the data used in its development and calibration. In the past, a large number of sediment transport equations were developed using data derived from laboratory experiments with shallow flows. When these equations are applied to natural rivers, especially to large rivers with deep flows, the predicted transport rates may vary drastically from measured values.

Large rivers are found throughout the world. These rivers are characterized not only by the large flow discharges, but also by their large flow depths. Average depths vary between 12 m and 68 m for Amazon River, and between 3 m and 22 m for Mississippi River (Posada, 1995). It is evident that, when compared with Amazon and Mississippi rivers, the flow depths used in laboratory flume experiments are very limited ($d < 0.5$ m). Correspondingly, under laboratory conditions Reynolds numbers are much smaller, Froude numbers are much larger, and water surface slopes are steeper than those conditions in large natural rivers. These limitations and differences also apply to data derived from shallow natural rivers such as Middle Loup River where depths vary from 0.25 m to 0.37 m (Colby and Hembree, 1955), and the Niobrara River where depths vary between 0.42 m and 0.58 m (Hubbell and Matejka, 1959). The dramatic differences experienced in flow depths, Reynolds numbers, Froude numbers, and water surface slopes between large rivers and laboratory flumes affect the resistance to flow as well as sediment wave movement and suspension, and consequently, the transport of sediment.

Due to the difficulties in obtaining accurate measurements in field, sediment transport data available in the literature are usually limited to those conducted in laboratory flumes. Complete and reliable sediment transport data obtained from field measurements are very limited. Therefore, it is not a surprise that most of the sediment transport equations largely rely on data from labora-

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tory experiments. Sediment transport is not a linear function of flow parameters such as depth, velocity, and slope. Using dimensionless transport parameters do not guarantee that an equation developed from laboratory conditions is automatically applicable to large river conditions if all similarity criteria are not satisfied simultaneously. Therefore it is not reasonable to extrapolate a transport equation developed largely from laboratory data to prototype conditions (especially large rivers) without any adjustment or modification. This limitation is neglected in the past by many researchers and applications. Recently, Posada and Nordin (1993) and Posada (1995) examined the Colby (1964) method and the Engelund and Hansen (1967) equation to estimate sediment loads in the deep flows of Amazon, Orinoco, and Mississippi rivers. Their analysis indicates that both methods overestimate the unit sediment discharges by almost a factor of two. The studies of Posada and Nordin (1993) and Posada (1995) point out to the deficiency in existing sediment transport equations to accurately reflect the sediment transport processes in large river conditions. Posada (1995) conducted a study which presented 149 sets of transport data for large rivers. Posada's large river data along with Toffaleti's (1968) large river data produce a reliable data base containing more than 400 measurements for the sediment transport in large rivers (see Table 1). In this paper, a new sediment transport equation for large rivers is developed by using the stream power concept and field measurements.

2 Data sources

The data used in this study are collected from medium and large alluvial rivers. A summary of the data is given in Table 1. In this study, large rivers refer to those with yearly average flow depths greater than 4 m, and medium rivers refer to those with yearly average flow depths between 2 m and 4 m. The large river data are used in the development of a new sediment transport relationship whereas the medium river data are used for verification. The large river data include the data from Amazon and Orinoco River Systems (Posada, 1995), Mississippi River System (Posada, 1995), Atchafalaya River at Simmesport, Louisiana (Toffaleti, 1968), Mississippi River at Tarbert Landing, Mississippi (Toffaleti, 1968), Mississippi River at St. Louis, Mississippi (Toffaleti, 1968), and Red River at Alexandria, Louisiana (Toffaleti, 1968). Bed-material loads for all large rivers are the measured suspended load plus the unmeasured load obtained from the Modified Einstein procedure (Colby and Hembree, 1955). The medium river data include ACOP Canal data of Mahmood et al.(1979), Chop Canal data of Chaudhry et al. (1970), Canal data of Chitale et al.(1966), Colorado River (US Bureau of Reclamation, 1958), river data of Leopold (1969), South American river and canal data of NEDCO (1973), Portugal River data of Peterson and Howells (1973), and Rio Grande River data of Nordin and Beverage (1965). The medium river data are all adopted from Brownlie (1981b). The data with flow depths smaller than 1.5m are excluded from the data base for the medium rivers. The flume data of Guy, Simons, and Richardson (1966), Stein (1965), Williams (1970) are also included in the data base to show the differences in sediment transport process between large rivers and experimental flumes.

As given in Table 1, the total numbers of data is 414 for large rivers, 535 for medium rivers. The flow discharges for large rivers are in the range of 134 m^3/s to 235000 m^3/s, flow velocities in the range of 0.21 m/s to 2.42 m/s, flow depths in the range of 3 m to 62.2 m, water surface slopes in the range of 0.02×10^{-4} to 1.8×10^{-4}, median bed material diameters in the range of 0.09 mm to 0.99 mm, and bed-material concentrations in the range of 0.1 ppm to 2360 ppm. The flow discharges for medium rivers are in the range of 13 m^3/s to 4791 m^3/s, flow velocities in the range of 0.20 m/s to 2.30 m/s, flow depths in the range of 1.50 m to 9.29 m, water surface slopes in the range of 0.06×10^{-4} to 25×10^{-4}, median bed material diameters in the range of 0.02 mm to 2.60 mm, and bed-material concentrations in the range of 3 ppm to 5759 ppm.

3 Sediment transport for shallow and deep flows

To show the difference of sediment transport between the large rivers and flume experiments, the values of bed-material concentrations are plotted against the dimensionless unit stream power in Fig. 1. The dimensionless unit stream power is defined as VS/ω_{50}, where, V is the flow velocity, S is the energy slope, and ω_{50} is the fall velocity corresponding to median size bed material. The product VS is termed as unit stream power (Yang, 1973; Bettess and White, 1987) and represents the stream power per unit weight of water ($=\gamma QLS/\gamma BdL$, where Q is the discharge, L is the length of the reach, B is the channel width, and γ is the specific weight of water). In Fig. 1, the flume data of Stein (1965), Guy, Simons, and Richardson (1966), and Williams (1970), and the large river data from Atchafalaya (Toffaleti 1968) and Mississippi, at St. Louis (Toffaleti, 1968) are presented. Flow depths are up to 0.37 m for flume experiments and from 4.66 m to 17.28 m for the large river data. Two different trend lines can be observed

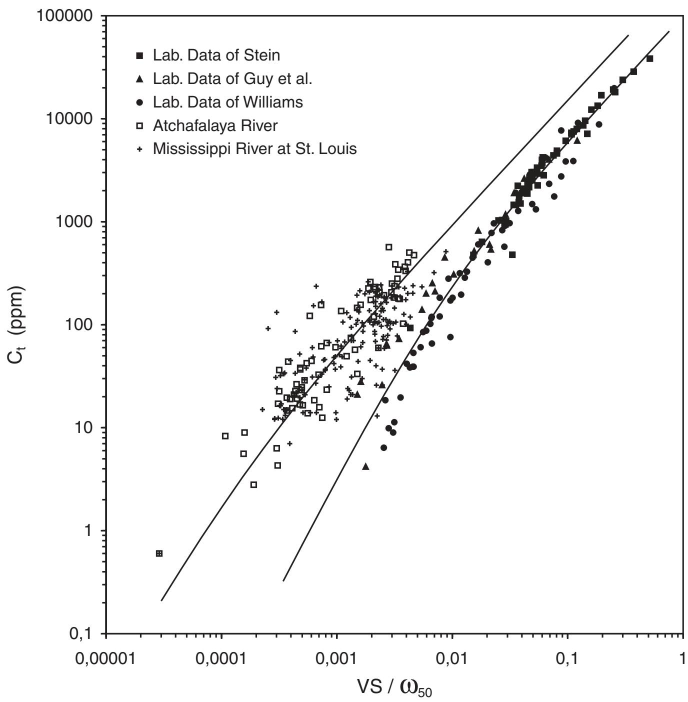

Fig. 1. The Variations of Bed-Material Concentration versus the Dimensionless Unit Stream Power.

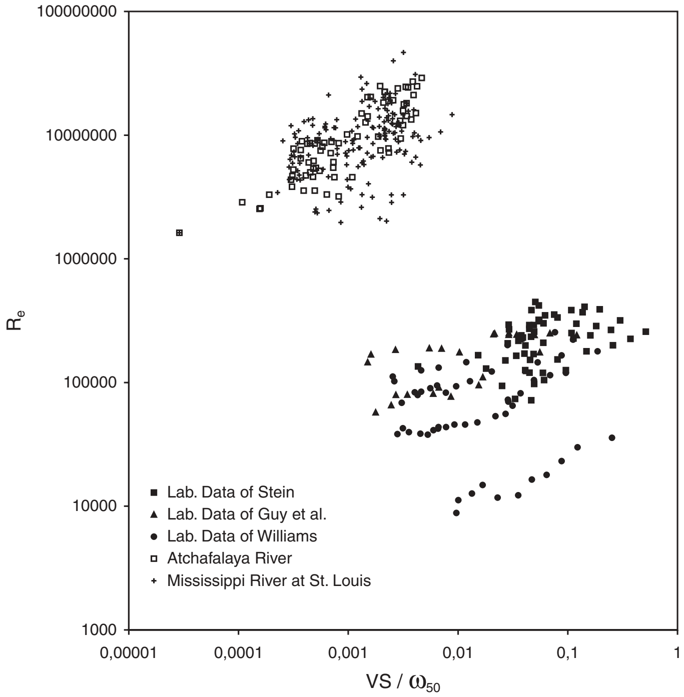

Fig. 2. The Variations of Reynolds Number versus the Dimensionless Unit Stream Power.

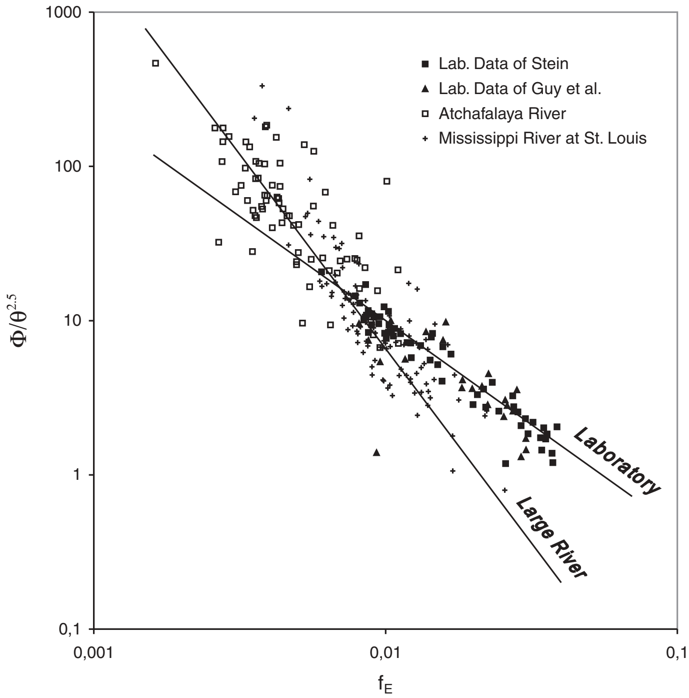

Fig. 3. The Relationship between $\Phi/\theta^{2.5}$ versus f_E.

for large river data and for flume data, respectively. For a given value of dimensionless unit stream power, VS/ω_{50}, large rivers are shown to have a higher bed-material concentration than the shallow flows.

In Fig. 2, the Reynolds number is plotted against VS/ω_{50} for laboratory flume and natural river conditions. It can be observed that the values of Reynolds numbers are in the range of 50,000 to 500,000 for flume data, and in the range of 2×10^6 to 1×10^8 for large rivers. The medium rivers lie between these two extremes. The difference between the values of Reynolds numbers in shallow-depth flume experiments and large rivers is about two orders of magnitude. This difference affects the resistance to flow and the associated sediment transport. This conclusion is further illustrated by the use of Engelund and Hansen's function. Engelund and Hansen's function (1967), which is one of the most widely used methods in practice due to its theoretical basis and confirmation with experiments, can be expressed as

$$\Phi / \theta^{2.5} = 0.1 / f_E \tag{1}$$

where Φ = dimensionless sediment transport parameter; θ = dimensionless shear stress; and f_E = friction parameter.

Fig. 3 shows the variation of $\Phi/\theta^{2.5}$ versus f_E for flume and large river data. It is obvious that sediment transport in flume experiments with depths less than 0.5 m follows a different relationship than large rivers with depths greater than 4m.

Since the range of flow depths in flume experiments are limited to shallow depths, the transport equations relying on mainly data derived from laboratory experiments do not adequately represent the variation of sediment transport in large rivers. This point is further illustrated in the comparison section through Figs. 5-7.

4 Energy concept for sediment transport

In developing a new transport relationship for large rivers, the stream power concept is used. The theoretical derivations based on energy and stream power concepts in the development of sediment transport equations include the gravitational theory by Velikanov (1954), the stream power theory by Bagnold (1966), and the unit stream power theory by Yang (1973) and Yang and Molinas (1982).

Gravitational Power Theory

Velikanov's gravitational power theory assumes that the power available in flowing water is equal to the sum of the power required to overcome flow resistance and the power required to keep sediment particles in suspension against the gravitational force. The general form of the Velikanov's equation can be expressed as

$$C_v = K \frac{V^3}{g\, d\, \omega} \tag{2}$$

where C_v = bed-material concentration by volume; d = flow depth; g = gravitational acceleration; ω = fall velocity; and K = parameter, which is related to Darcy-Weisbach resistance parameter by

$$K = \frac{s_g}{s_g - 1} \frac{f - f_0}{8} \tag{3}$$

where f_0, f = Darcy-Weisbach friction parameters for clear water and sediment-laden flow, respectively; and s_g = relative density. Following Velikanov's gravitational energy theory, Zhang (1959) and Dou (1974) derived the following transport equations, respectively,

$$C_t = K_1\left(\frac{V^3}{g\,R\omega}\right)^m \tag{4}$$

$$C_t = K_2\,\frac{V^3}{g\,d\omega} \tag{5}$$

where C_t = bed-material concentration by dry weight; m = exponent; R = hydraulic radius; and K_1, K_2 = parameters, which can be expressed as

$$K_1 = s_g\left(\frac{1}{s_g - 1}\;\frac{f_0 - f}{8\,k_1}\right)^{1/\alpha} \tag{6}$$

$$K_2 = \frac{s_g}{s_g - 1}\;\frac{k_2\;g}{C^2} \tag{7}$$

where C = Chézy roughness coefficients; k_1, k_2 = coefficients; and α = coefficient.

Stream Power Theory

Bagnold's stream power theory stated that, from the general physics, the available power of flow supplies the energy for the transport of sediment. From the energy concept, Bagnold derived an equation for bed load and suspended load respectively. The bed-material load is the sum of bedload and suspended load, i.e.

$$q_t = q_b + q_s = \frac{1}{s_g - 1}\,\tau_0 V\left(\frac{e_b}{\tan\alpha} + 0.01\frac{V}{\omega}\right) \tag{8}$$

where e_b = bed load transport efficiency; q_t, q_b, q_s = unit bed-material discharge, unit bedload discharge, and unit suspended load discharge, respectively; $\tan\alpha$ = ratio of tangential to normal shear force; and τ_0 = bed shear stress.

Unit Stream Power Theory

The rate of energy dissipation used in transporting sediment is related to the rate of sediment being transported (Yang, 1973). The relationship between concentration and unit stream power can be derived from well-established theories in turbulence and fluid mechanics (Yang and Molinas, 1982). Based on turbulent energy theory and Rouse's (1937) vertical sediment distribution equation, Yang and Molinas (1982) showed that the vertical sediment concentration distribution is related to the vertical distribution of rates of turbulent energy production, i.e.,

$$\frac{\overline{C}}{\overline{C}_a} = \left[\frac{\tau_{xy}\,\dfrac{d\,\bar{u}_x}{dy}}{\left(\tau_{xy}\,\dfrac{d\,\bar{u}_x}{dy}\right)_{y=a}}\right]^{Z_1} = \left[\frac{\overline{u_x'\,u_y'}\,\dfrac{d\,\bar{u}_x}{dy}}{\left(\overline{u_x'\,u_y'}\,\dfrac{d\,\bar{u}_x}{dy}\right)_{y=a}}\right]^{Z_1} \tag{9}$$

where $\overline{C}$, $\overline{C}_a$ = time-averaged sediment concentration at distances of y and a above the bed, respectively; $\bar{u}$, u' = mean and fluctuating parts of velocity, respectively; τ_{xy} = turbulent shear stress at distance y above the bed; $Z_1 = \beta Z$; β = coefficient; and

$$Z = \frac{\omega}{\kappa\,U_*} \tag{10}$$

where κ = von Kármán's universal coefficient; and $U_* = \sqrt{gdS}$ = shear velocity. It can be shown that (Yang and Molinas 1982)

$$C_t = M\left(\frac{VS}{\omega_{50}}\right)^N \tag{11}$$

where M, N = parameters. In Eq. (11), the term VS/ω_{50} is known as the dimensionless unit stream power.

5 Transport equation for large rivers

According to Einstein (1950), sediment transport is related to the energy slope due to grain resistance, S', rather than the total energy slope, S. Using S' in Eq. (11) and including submerged weight, the dimensionless unit stream power can be expressed as

$$C_t = M\left(\frac{V\,S'}{(s_g - 1)\;\omega_{50}}\right)^N \tag{12}$$

The strong correlation between C and dimensionless unit stream power is supported by laboratory data and some small river data (Yang, 1973; Yang and Molinas, 1982; Yang and Kong, 1991; Yang, Molinas, and Wu, 1996). Due to the uncertainty and inaccuracy in measuring the water surface slope in natural rivers, it is a very common practice to replace the slope term in Eqs.(11) and (12) with an appropriate velocity term. This transformation was strongly recommended by Simons and Şentürk (1992) for field applications. As stated by Simons and Şentürk, the principal reason for this suggestion relates to the difficulties involved in accurately measuring stream gradient, the simplicity and accuracy with which velocity and velocity distribution can be measured and/or estimated and the close correlation between energy gradient and velocity. To replace the energy slope term in Eq. (12) with a velocity term, the Darcy-Weisbach equation may be used, that is

$$S' = \frac{f' \, V^2}{8 \; g \; d} \tag{13}$$

where f' = Darcy-Weisbach friction factor corresponding to grain resistance. Substituting the slope in Eq. (12) with Eq. (13), we get

$$C_t = M \left(\frac{f' \, V^3}{(s_g - 1) \; g \; d \, \omega_{50}} \right)^N \tag{14}$$

The friction factor f' can be obtained from the universal logarithmic type vertical velocity distribution (Simons and Şentürk, 1992). The depth integrated velocity for wide channels is

$$\sqrt{\frac{8}{f'}} = \frac{V}{U'_*} = 5.75 \log_{10}\left(\frac{d}{k_s}\right) + C = 5.75 \log_{10}\left(\frac{\alpha d}{k_s}\right) \tag{15}$$

where C, α = coefficients; k_s = height of equivalent sand roughness; and U'_* = shear velocity corresponding to grain resistance. Substituting the expression for f' from Eq. (15) into Eq. (14), replacing k_s with nD_{50}, and rearranging terms give

$$C_t = M \left(\frac{V^3}{(s_g - 1)\, gd\, \omega_{50} \left[\log_{10}\left(\frac{\alpha d}{n D_{50}} \right) \right]^2} \right)^N \tag{16}$$

where n = coefficient.
Since $\log(\alpha/n)$ is much smaller than $\log(d/D_{50})$, neglecting this term in Eq. (16) results in

$$C_t = M \, \Psi^N \tag{17}$$

where Ψ = universal stream power, which is defined by

$$\Psi = \frac{V^3}{(s_g - 1)\, gd\, \omega_{50} \left[\log_{10}\left(\frac{d}{D_{50}} \right) \right]^2} \tag{18}$$

The relationship given by Eq. (16) is similar to those of Velikanov, Zhang, and Dou given by Eqs. (2), (4), and (5). Similar to Eq. (16), Eqs. (2), (4), and (5) are theoretically related to the resistance to flow through K, K_1, and K_2. However, in these relationships, K, K_1, and K_2 were treated as constant coefficients to be determined from measured data rather than parameters related to flow conditions.
Slope, as a term in the determination of sediment concentration or transport rate, is not preferred in practice. But neglecting the variation of resistance parameters as done in Eqs. (2), (4), and (5) is not correct either. Yang and Kong (1991) argued that the inaccuracy introduced into a transport equation by using a resistance equation to replace the energy slope may far exceed the inaccuracy caused by slope measurements. This comment is applicable to formulations which totally neglect the effects of resistance to flow on sediment transport. Additionally, this comment is applicable to conditions where relatively accurate slope measurements can be obtained (e.g., laboratory experiments). In large rivers where water surface slopes are extremely flat (see Table 1), the energy slope S is in the order of 10^{-5}. For this range of slope values, field measurements are highly sensitive to measurement errors. Therefore, for large rivers, expressing slope as a function of velocity and depth, and using an appropriate resistance equation for estimating the resistance parameter is a valid compromise in the determination of transport rate.
The values of C_t are plotted against Ψ in Fig. 4 for the 414 sets of large river data given in Table 1. This figure shows a close correlation between C_t and Ψ. The relationship between C_t and Ψ is nonlinear since M and N are not constants. Since the objective of this paper is to develop an equation which is applicable for large rivers with large Reynolds numbers, Eq. (17) is calibrated using only large river data given in Table 1. The resulting expression for Eq. (17) is

$$C_t \; (ppm) = \frac{1430 \; (0.86 + \sqrt{\Psi}\,) \; \Psi^{1.5}}{0.016 + \Psi} \tag{19}$$

where the definition of Ψ is given by Eq. (18), and the fall velocity is computed using Rubey's (1933) method.

6 Comparison

There are numerous well-known bed-material load equations in literature to estimate sediment transport in alluvial rivers. Com-

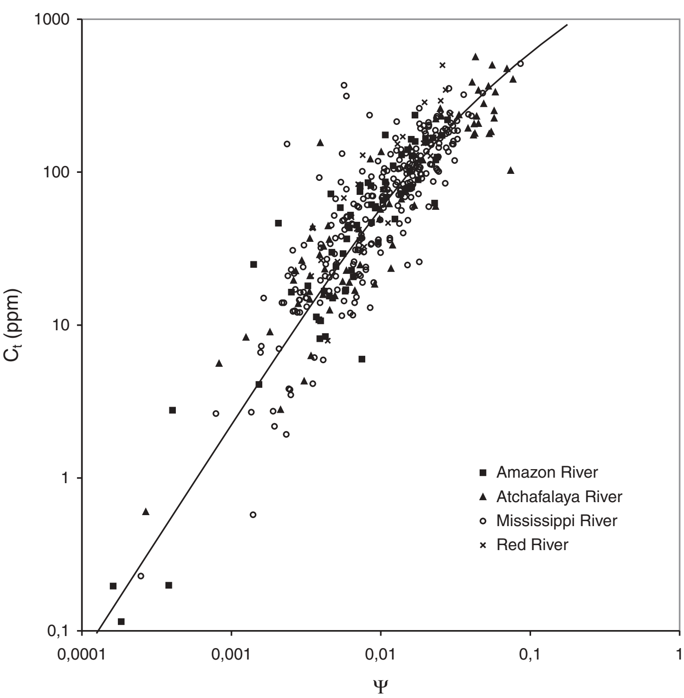

Fig. 4. The Relationship between Bed-material Concentration and the Universal Stream Power.

Table 1. Summary of Laboratory and River Data

Data Source (1)	Flow Discharge (m^3/s) (2)	Flow Velocity (m/s) (3)	Flow Depth (m) (4)	Water Surface Slope×10^4 (5)	Median Diameter (mm) (6)	Bed-Material Concentration (ppm) (7)	No. of Data (8)
(a) Large Rivers ($\overline{d}_{yr}$ (i) > 4.0 m)							
Amazon and Orinoco River Systems (Posada 1995)	134-235000	0.37-2.42	3.56-62.33	0.14-1.8	0.093-0.90	0.1-2360	65
Mississippi River System (Posada 1995)	332-4100	0.37-1.77	3.17-21.80	0.03-1.8	0.18-0.99	0.2-370	84
Atchafalaya River at Simmesport (Toffaleti 1968)	382-14188	0.21-2.03	6.10-14.75	0.02-0.51	0.091-0.31	0.6-570	72
Mississippi River at Tarbert Landing (Toffaleti 1968)	4248-28830	0.62-1.61	6.74-16.40	0.18-0.43	0.18-0.33	12-260	53
Mississippi River at St. Louis (Toffaleti 1968)	1512-21608	0.62-2.42	4.66-17.28	0.25-1.34	0.18-1.15	7-510	111
Red River at Alexandria (Toffaleti 1968)	190-1538	0.37-1.14	3.00-7.38	0.66-0.82	0.10-0.22	8-500	29
Total of Large Rivers	134-235000	0.21-2.42	3.00-62.33	0.02-1.8	0.091-0.99	0.1-2360	414
(b) Medium Rivers (2.0 m < $\overline{d}_{yr}$ < 4.0 m, d > 1.5 m)							
ACOP Canal Data of Mahmood et al. (1979)(ii)	30-529	0.50-1.30	1.50-4.30	0.55-1.7	0.083-0.36	17-2083	131
Chop Canal Data of Chaudhry et al. (1970)(ii)	28-428	0.69-1.60	1.68-3.41	0.51-2.5	0.10-0.31	116-1317	32
Canal Data of Chitale et al. (1966)(ii)	13-242	0.51-1.06	1.57-3.56	0.57-1.2	0.02-0.082	512-5759	28
Colorado River (US Bureau of Reclamation, 1958)(ii)	97-500	0.53-1.27	1.51-3.89	0.37-4.1	0.16-0.70	18-769	91
River Data of Leopold (1969)(ii)	109-499	0.56-1.26	1.50-4.11	0.37-3.5	0.14-0.81	11-564	54
South American River and Canal Data of NEDCO (1973)(ii)	29-4791	0.20-1.64	1.53-9.29	0.06-6.2	0.10-1.08	3-3000	73
Portugal River Data of Peterson and Howells (1973) (ii)	107-660	0.93-1.44	1.50-2.44	6.1-9.7	2.20-2.60	54-351	109
Rio Grande River Data of Nordin and Beverage (1965) (ii)	79-286	1.25-2.30	1.50-3.12	13-25	0.31-1.91	1300-5310	16
Total of Medium Rivers	13-4791	0.20-2.30	1.5-9.29	0.06-25	0.02-2.60	3-5759	534
(c) Laboratory Flumes							
Guy, Simons, and Richardson (1966)	0.15-0.64	0.41-1.85	0.12-0.34	3.7-130	0.93	4-10300	31
Stein (1965)	0.078-0.48	0.42-1.84	0.09-0.37	6.1-170	0.40	93-39310	57
Williams (1970)	0.003-0.16	0.37-1.87	0.03-0.22	6-330	1.35	6-20000	83
Total of Laboratory Flumes	0.003-0.64	0.37-1.87	0.03-0.37	3.7-330	0.40-1.35	4-39310	171

Note: (i) Yearly average flow depth; (ii) Data from Brownlie (1981b).

parison and evaluation of transport equations can be found in White, et al. (1975), Alonso (1980), Brownlie (1981a), American Society of Civil Engineers (1982), Yang and Molinas (1982), Vetter (1988), and Yang and Wan (1991). In this study, the commonly recommended sediment transport equations of Engelund and Hansen (1967), Ackers and White (1973), and Yang (1973) are compared with the proposed sediment transport equation for large rivers using the universal stream power.

Engelund and Hansen (1967) applied Bagnold's (1966) stream power concept and the similarity principle to obtain an expression for estimating sediment concentration. Engelund and Hansen's equation was developed based on 116 sets of flume data of Guy et al. (1966). These flume data include sediment diameters of 0.19, 0.27, 0.45, and 0.93 mm with flow depths up to 0.34 m. Ackers and White developed their transport functions based on the mobility theory and Bagnold's (1966) stream power concept. The relationships for A, C, m, and, n were obtained based on 925 laboratory experiments for depths of flow up to 0.4 m, and for particle sizes ranging from 0.04 mm to 4.0 mm. Yang's (1973) sediment transport formula is based on unit stream power theory. Yang's equation was derived from 463 sets of laboratory data ranging from 0.137 mm to 1.35 mm for particle size, and 0.022 m to 0.86 m for flow depth. These three equations were all developed based on energy concept. Toffaleti's (1968) method is also included in the comparison because of the use of large river data in its development. Toffaleti's (1968) approach for the determination of bed-material load is based on the concept of Einstein (1950) and Einstein and Chien (1953). Toffaleti assumed a thin

Table 2. Summary of Comparison between Computed and Measured Bed-Material Concentrations for Large Rivers.

Author of Formula (1)	Data in Range of Discrepancy Ratio, R_i (i) (%)					Mean Normalized Error, MNE (iii) (%)	Correlation Coefficient, R(iv)	No. of Data, N
	0.75-1.25 (2)	0.5-1.5 (3)	0.25-1.75 (4)	0.5-2.0 (5)	$\bar{R}$ (ii) (6)	(7)	(8)	(9)
Proposed Eq. (19)	41.6	70.3	85.0	78.0	1.14	50.3	0.81	414
Engelund and Hansen	23.0	46.9	68.4	58.4	2.21	164.9	0.58	414
Ackers and White	30.4	52.9	71.3	62.1	1.65	108.0	0.25	414
Yang	10.6	26.2	55.2	27.1	0.63	84.3	0.49	414
Toffaleti	33.1	62.8	80.2	71.7	1.20	62.2	0.72	414

Table 3. Summary of Comparison between Computed and Measured Bed-Material Concentrations for Medium Rivers

Author of Formula (1)	Data in Range of Discrepancy Ratio, R_i (i) (%)					Mean Normalized Error, MNE (iii) (%)	Correlation Coefficient, R(iv)	No. of Data, N
	0.75-1.25 (2)	0.5-1.5 (3)	0.25-1.75 (4)	0.5-2.0 (5)	$\bar{R}$ (ii) (6)	(7)	(8)	(9)
Proposed Eq. (19)	27.9	56.2	77.0	62.9	1.19	70.6	0.77	534
Engelund and Hansen	24.7	46.2	71.5	56.2	1.21	73.3	0.55	534
Ackers and White	15.7	36.1	70.6	41.0	4.50	427.0	0.17	534
Yang	7.9	22.1	46.0	30.2	0.89	86.8	0.56	534
Toffaleti	8.6	18.4	40.1	21.0	0.60	84.6	0.65	534

Notes: (i) $R_i = C_{tci} / C_{tmi}$

(ii) $$\bar{R} = \frac{1}{N}\sum_{i}^{N} C_{tci} / C_{tmi} = \frac{1}{N}\sum_{i}^{N} R_i$$

(iii) $$MNE = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{C_{tci} - C_{tmi}}{C_{tmi}}\right|$$

(iv) $$R = \frac{\sum_{i=1}^{N}(C_{tci} - \bar{C}_{tc})(C_{tmi} - \bar{C}_{tm})}{\sqrt{\sum_{i=1}^{N}(C_{tci} - \bar{C}_{tc})^2 \sum_{i=1}^{N}(C_{tmi} - \bar{C}_{tm})^2}}$$

bed layer in which the bed load transport takes place. This layer provides a reference concentration for the suspended load computation. He divided the depth, d, into four zones: the bed zone, the lower, middle and upper zones, and presented relative thickness, velocity distribution and sediment concentration for each zone (Toffaleti, 1968 and 1969). This method was suggested for large sand-bed rivers by Simons and Şentürk (1992), and Shen (1979), and Yang (1996).

The data from large and medium rivers given in Table 1 are used in the comparison. Tables 2-3 summarize the comparisons of the accuracy of the equations used in the analysis. The discrepancy ratio, mean discrepancy ratio, mean normalized error, and correlation coefficient between the computed and measured bed-material concentrations are used to indicate the accuracy of each equation. The discrepancy ratio, R_i, is defined as the ratio between computed and measured values. The mean discrepancy ratio, $\bar{R}$, is expressed as the average value of discrepancy ratios. The mean normalized error, MNE, is defined as the average value of absolute relative error between computed and measured concentrations.

For 414 sets of large river data compared, the proposed equation (19) and the Toffaleti method give mean discrepancy ratios of 1.14 and 1.20, respectively. The Engelund and Hansen formula and the Ackers and White formula give mean discrepancy ratios of 2.21 and 1.65, respectively. This indicates that these two methods, on the average, overestimate the sediment transport rate. A similar conclusion for the Engelund and Hansen formula was obtained by Posada (1995). Posada found that Engelund and Hansen's formula overpredicts the sediment transport rate by a factor of about two for deep rivers. The mean discrepancy ratio corresponding to the Yang formula is 0.63. This indicates that this formula underpredicts the sediment transport rate in large rivers. The discrepancy ratios listed in Table 2 show the distribution of accuracy for each equation. The values of discrepancy ratios indicate that Eq. (19) ranks at the top of the five equations in all ranges. The mean normalized errors are 50.3, 164.9, 108.0, 84.3, and 62.2; the correlation coefficients are 0.81, 0.58, 0.25, 0.49, and

0.72 for Eq. (19), the Engelund and Hansen, Ackers and White, Yang, and Toffaleti formulas, respectively. Both the mean normalized errors and the correlation coefficients indicate that the best agreement between the computed and measured values is given by Eq. (19). The Toffaleti method also gives reasonable predictions for large rivers. For large river data, Ackers and White equation shows poor performance using correlation coefficient as performance criteria. On the other hand, using the discrepancy ratios criteria, it can be seen that it outperforms Yang equation in every range. Our review of data indicated that the Amazon river data was the main reason for the poor correlation coefficient performance of Ackers and White equation. When these data were excluded, the correlation coefficient became 0.72; other statistics remained relatively the same.

To validate the applicability of the proposed sediment transport equation (19), an independent test is needed. Due to the limited number of available large river data, rather than dividing the 414 sets of large river data into subsets for comparison, the 534 sets of medium river data given in Table 1 are used as an alternative for this purpose. A summary of comparison between the computed and measured bed-material concentrations is given in Table 3. The mean discrepancies for the proposed equation, the Engelund and Hansen, Ackers and White, Yang, and Toffaleti equations are 1.19, 1.21, 4.50, 0.89, and 0.60, respectively; the mean normalized errors are 70.6, 73.3, 427.0, 86.8, and 84.6, respectively; the correlation coefficients are 0.77, 0.55, 0.17, 0.56, and 0.65, respectively. Also, for the proposed equation, the Engelund and Hansen, Ackers and White, Yang, and Toffaleti equations 77.0%, 71.5%, 70.6%, 46.0%, and 40.6% of the data points lie between the discrepancy ratios of 0.25-1.75, respectively. These statistical results confirm that the proposed equation ranks as the best predictor. The Engelund and Hansen ranks at the second-best predictor for medium rivers. The performance of the Toffaleti formula for medium river data that were not included in its development is poor.

The comparisons between computed and measured bed-material

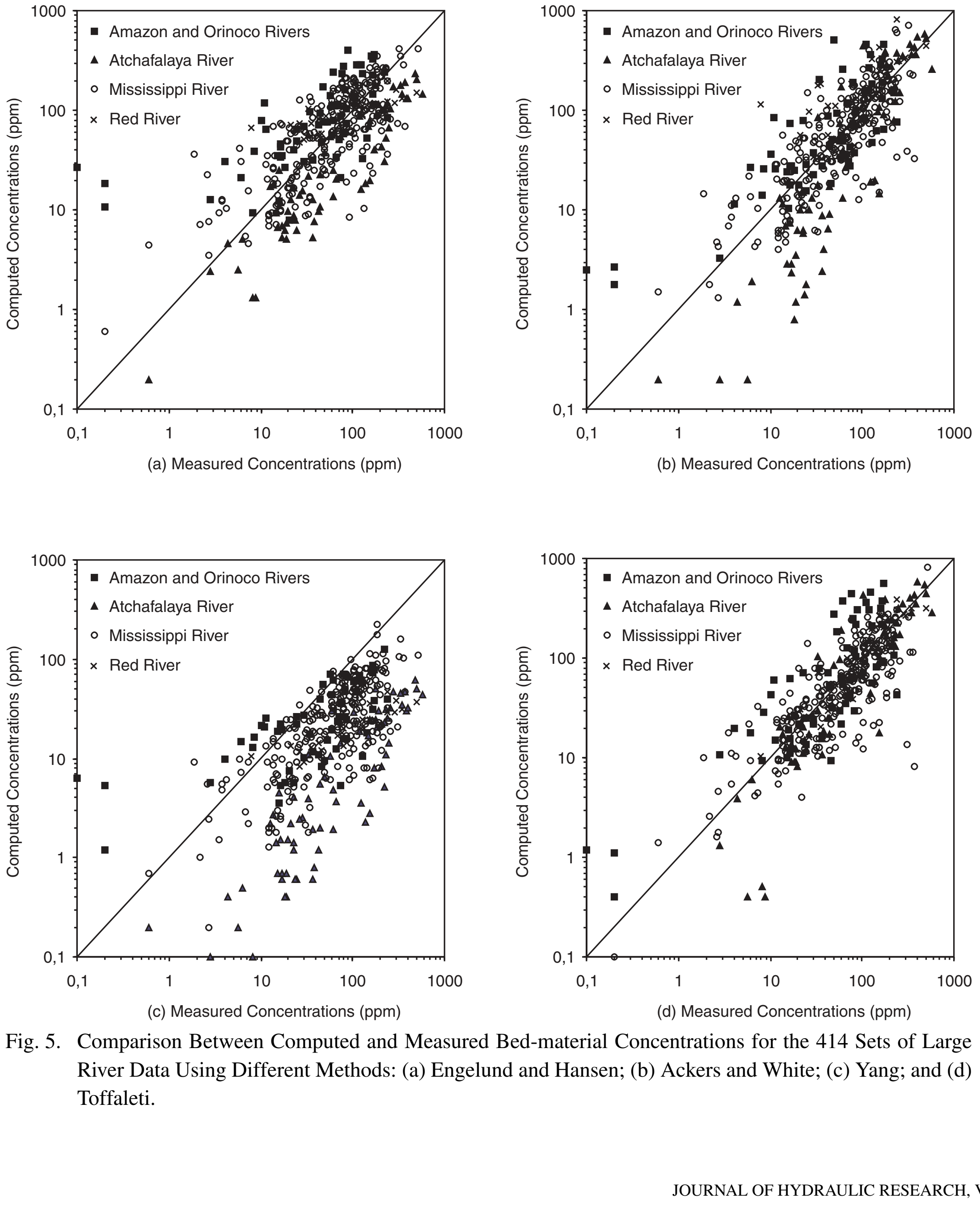

Fig. 5. Comparison Between Computed and Measured Bed-material Concentrations for the 414 Sets of Large River Data Using Different Methods: (a) Engelund and Hansen; (b) Ackers and White; (c) Yang; and (d) Toffaleti.

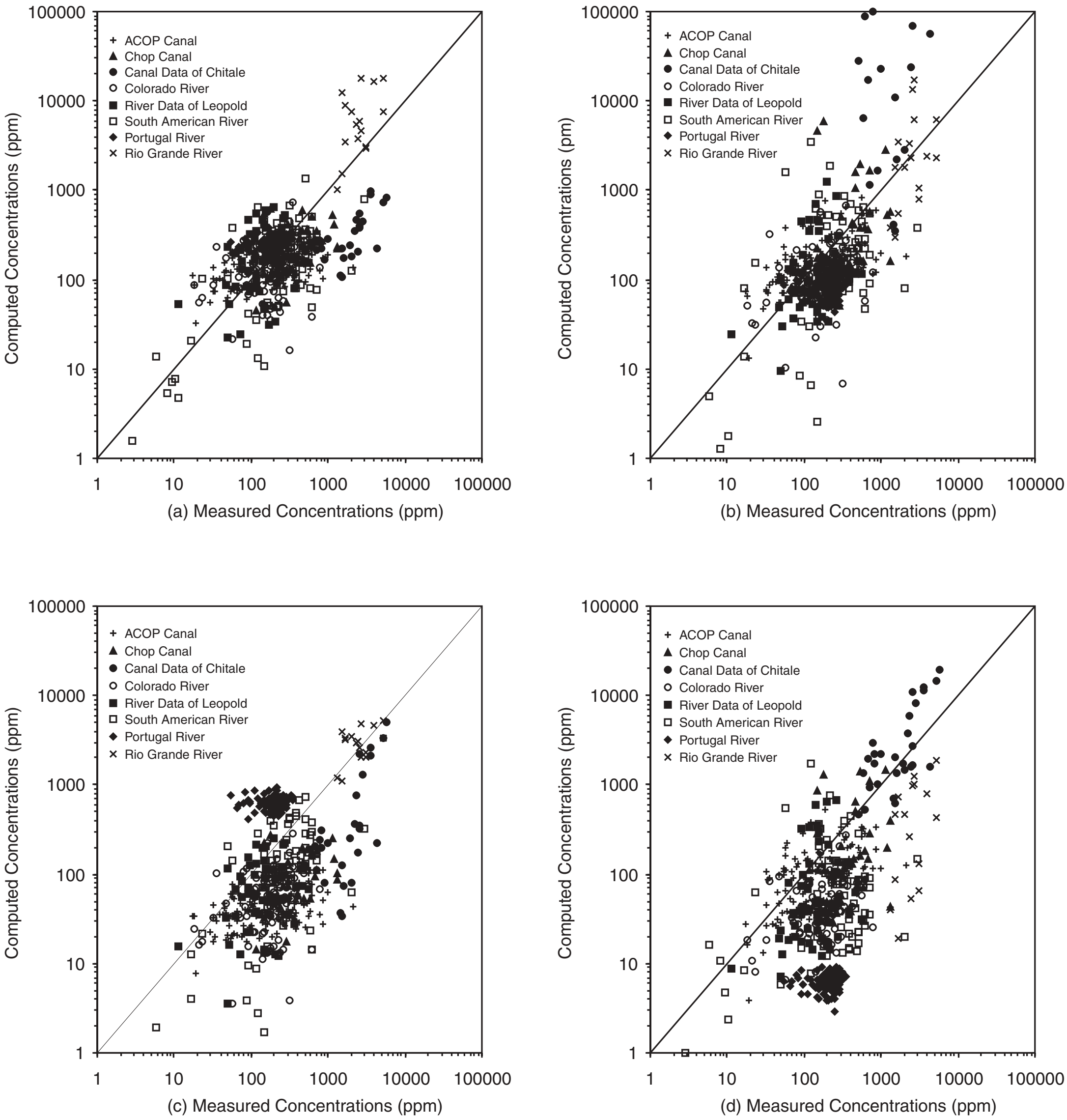

Fig. 6. Comparison Between Computed and Measured Bed-material Concentrations for the 534 Sets of Medium River Data Using Different Methods: (a) Engelund and Hansen; (b) Ackers and White; (c) Yang; and (d) Toffaleti.

concentrations for the Engelund and Hansen, Ackers and White, Yang, and Toffaleti equations are plotted in Fig. 5 for the 414 sets of large river data, and in Fig. 6 for the 534 sets of medium river data. Fig. 7 is the results by the use of Eq. (19) and shows that the computed bed-material concentrations are in close agreement with the measurements for both large and medium rivers.

7 Conclusions

In this study, the sediment transport in large rivers with deep flows are investigated. A new sediment transport equation (19) is developed for large rivers in terms of the universal stream power defined by Eq. (18). This proposed equation is compared with selected bed-material load equations using large and medium river data. The following conclusions are reached:

1. Equations developed using mainly flume experiments with shallow flows can not represent the sediment transport taking place in large rivers with yearly average flow depth great than 4m. The use of dimensionless homogeneous parameters in an equation is not sufficient to ensure its applicability to flow conditions where flow depths are several orders of magnitude larger.

2. The Engelund and Hansen, Ackers and White, and Yang formulas developed mainly from flume experiments with shallow flows ($d < 0.5$ m) are not applicable to large rivers (yearly average flow depth great than 4 m), while the Toffaleti's method developed mainly from large rivers gives fairly well predictions of sediment transport rates. The results for the Toffaleti method confirms the suggestions made by Simons and Şentürk (1992), and Shen (1979), and Yang (1996) for the selection of the Toffaleti method over Engelund and Hansen, Ackers and White, and Yang formulas for large sand-bed rivers. For medium rivers (yearly average flow depth between 2 m and 4 m), Engelund and Hansen formula gives reasonable predictions of

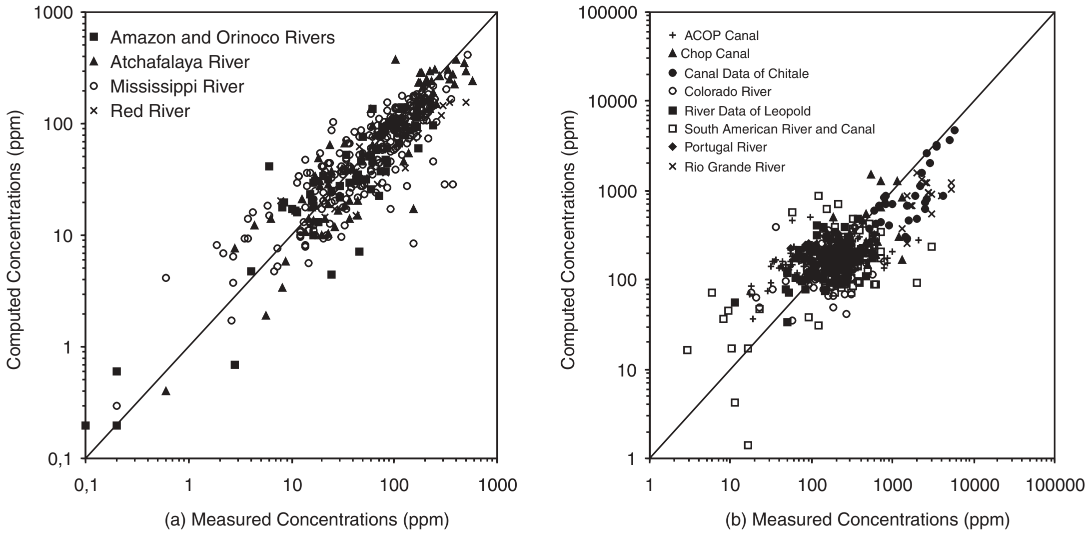

Fig. 7. Comparison Between Computed and Measured Bed-material Concentrations Using the Proposed Eq. (19): (a) For the 414 Sets of Large River Data; (b) For the 534 Sets of Medium River Data.

sediment transport rates, while the Ackers and White formula overpredicts and Yang and Toffaleti formulas underpredict the transport rates.

3. The statistical analysis using 414 sets of large river data and 534 sets of medium river data show that the proposed sediment transport equation (19) based on universal stream power is the most accurate predictor for estimating the bed-material concentration in large and medium sand-bed rivers.

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Notations

The following symbols are used in this paper:

C	=	coefficient;
C	=	Chézy roughness coefficient;
C_t	=	bed-material concentration by dry weight;
C_v	=	bed-material concentration by volume;
$\overline{C}$, $\overline{C}_a$	=	time-averaged sediment concentration at distance of y and a above the bed, respectively;
D	=	particle size of bed material;
D_{50}	=	particle size for which 50 percent of bed material is finer by weight;
d	=	flow depth;
e_b	=	bed load transport efficiency;
f_0, f	=	Darcy-Weishbach friction parameters for clear water and sediment-laden flow, respectively;
f_E	=	friction parameter used by Engelund and Hansen;
f'	=	Darcy-Weishbach friction parameter corresponding to grain resistance;
g	=	gravitational acceleration;
K, K_1, K_2	=	Coefficients;
k_1, k_2	=	Coefficients;
k_s	=	height of equivalent sand roughness;
M, N	=	parameters;
m, n	=	coefficients;
q_t, q_b, q_s	=	unit bed-material discharge, unit bed load discharge, and unit suspended load discharge, respectively;
R	=	hydraulic radius;
R	=	correlation coefficient;
R_i	=	discrepancy ratio between computed and measured values;
$\overline{R}$	=	average value of discrepancy ratios;
S	=	energy slope;
S'	=	energy slope corresponding to grain resistance;
s_g	=	relative density = ρ_s/ρ;
$\overline{u}$, u'	=	mean and fluctuating components of velocity, respectively;
U_*	=	shear velocity = $\sqrt{gdS}$;
U'_*	=	shear velocity corresponding to grain resistance;
V	=	flow velocity;
y	=	distance above river bed;
Z	=	Rouse number = $\omega / \kappa U_*$;
Z_1	=	Modified Rouse number = βZ;
α, β	=	coefficients;

γ, γ_s = specific weight of water and sediment, respectively;
θ = dimensionless shear stress;
κ = von Karman's universal coefficient;
τ_0 = bed shear stress;
τ_{xy} = turbulent shear stress at distance y above the bed;
Φ = dimensionless sediment transport parameter;
Ψ = universal stream power;
ω = fall velocity of sediment corresponding to particle size D; and
ω_{50} = fall velocity of sediment corresponding to particle size D_{50}.