Hydraulic condition for undular-jump formations

Hydraulique de la formation du ressaut ondule

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ABSTRACT

This paper presents the upper limit of the inflow Froude number for undular-jump formations in smooth rectangular channels. It has been found that the formation of undular jumps depends not only on the inflow Froude number but on the boundary-layer development at the toe of the jump under conditions in which the effects of the aspect ratio and the Reynolds number on the flow condition are negligible. The velocity of the first wave crest immediately before the breaking is at a maximum near the water surface and becomes a critical velocity. For the undular jumps with the developing inflow, the upper limit of the Froude number F_{1limit} has been shown experimentally as F_{1limit} = 1.3-2.3. For the fully developed inflow, $F_{1limit} \fallingdotseq 1.7$ has also been obtained, and it shows the same value as described in many textbooks. The upper limit of the inflow Froude number for undular-jump formations has been derived by taking account of the boundary-layer development and considering the flow along the water surface immediately before the breaking. The predicted values agree with experimental results.

RÈSUMÈ

This paper presents the upper limit of the inflow Froude number for undular-jump formations in smooth rectangular channels. It has been found that the formation of undular jumps depends not only on the inflow Froude number but on the boundary-layer development at the toe of the jump under conditions in which the effects of the aspect ratio and the Reynolds number on the flow condition are negligible. The velocity of the first wave crest immediately before the breaking is at a maximum near the water surface and becomes a critical velocity. For the undular jumps with the developing inflow, the upper limit of the Froude number F_{1limit} has been shown experimentally as F_{1limit} = 1.3-2.3. For the fully developed inflow, $F_{1limit} \fallingdotseq 1.7$ has also been obtained, and it shows the same value as described in many textbooks. The upper limit of the inflow Froude number for undular-jump formations has been derived by taking account of the boundary-layer development and considering the flow along the water surface immediately before the breaking. The predicted values agree with experimental results.

Introduction

The undular jump is formed for low supercritical- inflow Froude numbers, and is characterized by undulations of the water surface without a surface roller [Chow (1959)].
The formation of undular jumps can be seen in a flood flow or a flow below a sluice gate or a weir, and undulations might cause bank erosion [Chanson (1995), Reinauer and Hager (1995)]. To study the characteristics of the undular jumps and to know the hydraulic conditions for the formation of the undular jumps are significant for the design and management of hydraulic structures. Because of the formation of undulations without a surface roller, undular jumps might be useful for planning water sports and recreational activities such as canoeing and rafting in rivers.
According to some generally known textbooks, the undular jump is formed if the value of the inflow Froude number is less than 1.7 [e.g., Chow (1959)]. Ippen and Harleman (1954), Iwasa (1955), and Hager and Hutter (1984) investigated the boundary between an undular jump and a jump with a surface roller from a theoretical approach, and they proposed the upper limit of the inflow Froude number for an undular-jump formation as $F_{1limit} \fallingdotseq$ 1.4-1.7.
Also, Bradley and Peterka (1957) experimentally showed the boundary as F_{1limit} = 1.7. Chanson and Montes (1995) reported that an undular-jump formation depends on the inflow Froude number and the aspect ratio, and that an undular jump is formed even if the value of the inflow Froude number F_1 is larger than 1.7 ($F_{1limit} \fallingdotseq$ 1.5-2.9). Considering the applicable range of the Froude similarity law, Reinauer and Hager (1995) proposed the boundary between an undular jump and a jump with a surface roller as F_{1limit} = 1.3-1.6.
By comparing the results of all of the researchers, it can be seen that general agreement on the upper limit of the Froude number has not been obtained.
Recently, a systematic investigation has shown that the flow condition of the undular jumps is governed by the inflow Froude number F_1 ($F_1 = v_1/\sqrt{gh_1}$; v_1= average velocity at the toe of the jump and h_1 = supercritical depth at the toe of the jump), the aspect ratio B/h_1 (B = channel width), and the turbulent boundary-layer development δ/h_1 [δ = thickness of the boundary layer at the toe of the jump (Fig.1)] as the fundamental hydraulic quantities [Ohtsu et al. (1995),(1996),(1997)]. Also, the effect of the Reynolds number on the flow characteristics of the undular jumps has been discussed, and the condition for the Froude similarity law has been clarified [Ohtsu et al. (1995), (1996), (1997)].
This paper shows that the upper limit of the Froude number for undular-jump formations F_{1limit} depends on the turbulent boundary-layer development at the toe of the jump under conditions in which the effects of the aspect ratio and the Reynolds number on the flow characteristics are negligible. For the formation of the undular jumps with the developing inflow, the upper limit of the Froude number has been shown experimentally as F_{1limit} = 1.3-

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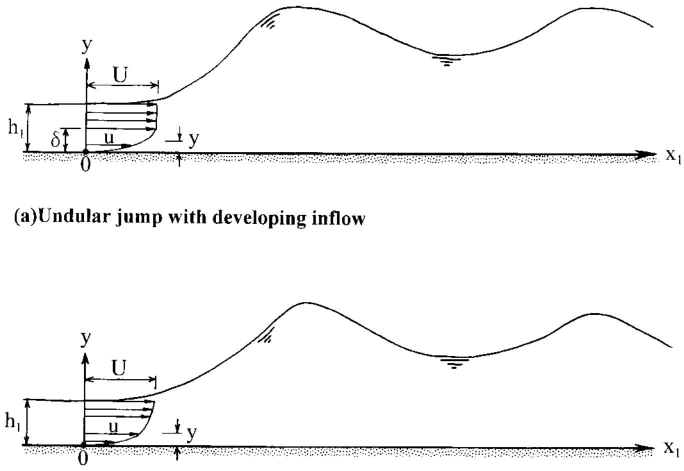


(a)Undular jump with developing inflow

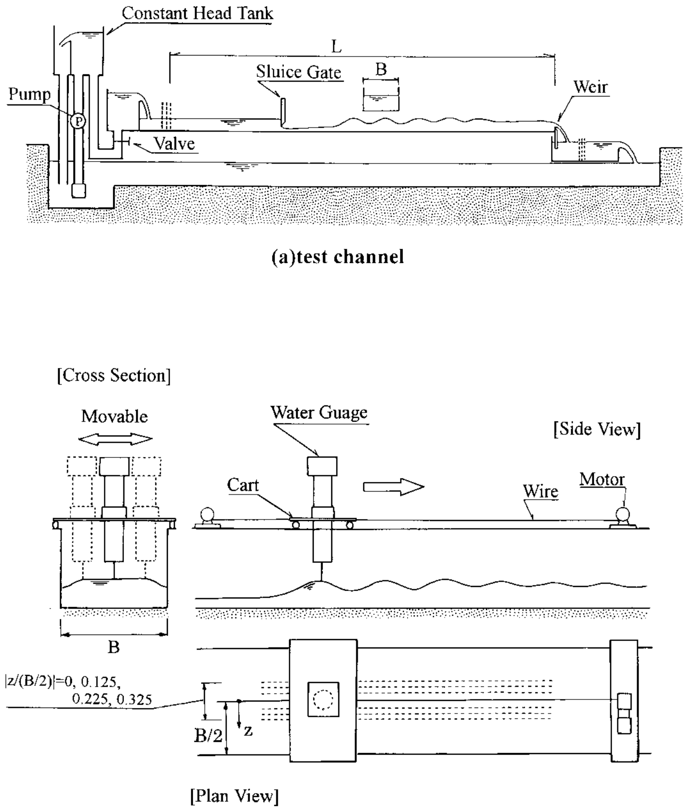
(b)Undular jump with fully developed inflow

Fig. 1. Turbulent boundary-layer development at toe of undular jump

2.3. For the fully developed inflow, the upper limit of the Froude number shows the same value ($F_{1limit} = \sqrt{3} \fallingdotseq 1.7$) as described in the well-known textbooks. Further, the flow immediately before the breaking has been characterized, and it can be found that the velocity at the first wave crest is at a maximum near the water surface, and that it becomes a critical velocity. Considering the flow along the water surface between the toe of the jump and the first wave crest immediately before the breaking, and taking account of the turbulent boundary-layer development at the toe of the jump, an equation for the upper limit of the inflow Froude number has been derived. The predicted results have been verified by laboratory experiments.

Experiments

Smooth rectangular horizontal channels with a sluice gate [Fig. 2 (a)] were used, and the investigation of the characteristics of undular jumps was made under a wide range of experimental conditions (Table 1). In order to know the water-surface profile of undular jumps accurately, a water gauge with a servomechanism [Ohtsu and Yasuda (1994)] was used. By moving the water gauge with a cart [see Fig. 2 (b)], the water surface can be measured continuously (movement speed = 0.114 m/s; sampling frequency = 50 Hz). As shown by Fig. 2 (b), an averaged water-surface profile was obtained by measuring the water surfaces of 7 longitudinal planes in the region of $0 \leq |z/(B/2)| \leq 0.325$ (z = transverse coordinate from the centerline of the channel). Further, the velocity in undular jumps was measured by using a one-dimensional Laser Doppler-Velocity meter (LDV) [sampling time = 164 s; sampling frequency = 25 Hz], a propeller-velocity meter with a 3-mm diameter [sampling time = 60 s; sampling frequency = 25 Hz], and a Plandtl-type Pitot tube.



(a)test channel

(b)water gauge with a servomechanism

Fig. 2. Experimental set-up

Hydraulic conditions for the formation of undular jumps

In the undular jump for $F_1 > 1.2$, as shown by Fig. 3, a shock wave is formed near the toe of the jump and develops from the channel sidewall [Ohtsu et al. (1995), Chanson and Montes (1995)]. If the shock wave does not cross upstream of the first wave crest, the main flow, except for the flow near the sidewall, is two-dimensional. Also, the effect of the aspect ratio B/h_1 on the flow characteristics is negligible [Ohtsu et al. (1996), (1997)]. For $Re \geq 65{,}000$ ($Re = v_1 h_1/\nu = q/\nu$; q = discharge per unit width and ν = kinematic viscosity), the effect of the Reynolds number on the flow conditions of the undular jumps is negligible [Ohtsu et al. (1995),(1996)].

Table 1 Experimental conditions

B(cm)	10.5	15.5	20	28	40	80
L(m)	5.0	5.0	17, 20	17	17, 20	14.5
h_1(cm)	2.63−3.39	3.93−5.17	2.21−11.71	8.00−10.77	2.20−11.75	3.67−10.66
$Q(m^3/s)\times10^3$	2.20−3.02	6.54−9.02	4.79−31.6	30.4−38.5	9.36−65.5	38.5−100
F_1	1.30−2.16	1.23−2.39	1.16−2.39	1.24−1.54	1.15−2.38	1.13−2.46
$Re\times10^4$	2.0−2.9	4.0−5.4	2.5−17.0	8.8−11.5	2.1−15.9	4.5−12.0
δ/h_1	1.0	1.0	0−1.0	0−1.0	0−1.0	0−1.0

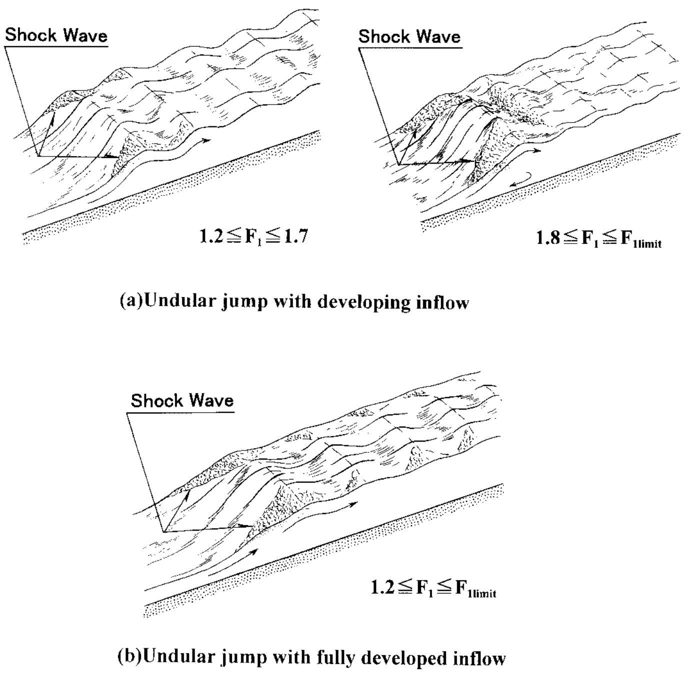


Fig. 3. Flow condition of undular jumps

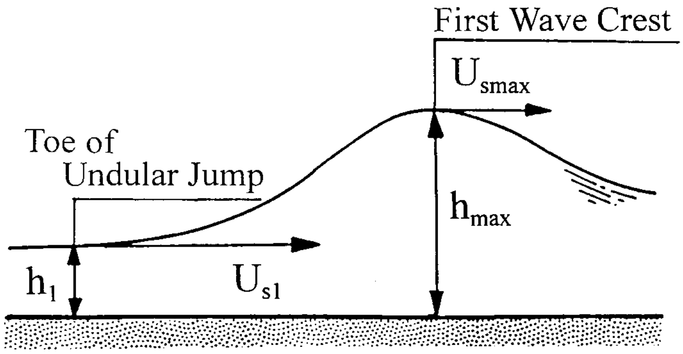


Fig. 4. Definition sketch of first wave in undular jump

The boundary between undular jump and jump with a surface roller (i.e., the upper limit of the Froude number for the formation of undular jumps F_{1limit}) depends on the turbulent boundary-layer development at the toe of the jump under conditions in which the effects of the aspect ratio and the Reynolds number on the flow characteristics are negligible. For a given inflow Froude number, the velocity near the water surface at the toe of the jump gives a different value according to the degree of boundary-layer development. Considering the streamline along the water surface between the toe of the jump and the first wave crest, the upper limit of the inflow Froude number for undular-jump formations has been obtained as follows:

Applying the Bernoulli theorem along the water surface between the toe of the jump and the first wave crest gives

$$\frac{U_{s1}^{2}}{2g}+h_1=\frac{U_{smax}^{2}}{2g}+h_{max} \tag{1}$$

where U_{s1} = velocity of the water surface at the toe of the jump; U_{smax} = velocity of the water surface at the first wave crest; and h_{max} = wave height at the first wave crest (Fig. 4).

The relationship between the averaged velocity v_1 and the velocity of the water surface at the toe of the jump U_{s1} can be expressed as

$$\mathbf{U_{s1}=kv_1} \qquad \left[\mathbf{v_1h_1=U_{s1}}\left\{\int_0^\delta\left(\frac{\mathbf{y}}{\delta}\right)^{\frac{1}{7}}\mathbf{dy}+(\mathbf{h_1}-\delta)\right\}\right] \tag{2}$$

where k is the correction coefficient and is given by (3), because the velocity distribution in the boundary layer at the toe of the jump is approximated by the one-seventh power law, as shown by Fig. 5. In Fig. 5, u is the mean velocity at y = y, U is the velocity outside the boundary layer (at y = δ), and y is the vertical coordinate from the channel bottom (Fig. 1).

$$k=\frac{1}{1-\dfrac{\delta}{8h_1}} \tag{3}$$

In addition, the value of k for $\delta/h_1 = 1$ (i.e., the fully developed inflow) is 8/7 from (3).

For the first wave crest of undular jumps, the wave height is approximated by (4) (the solid line in Fig. 6) [Ohtsu et al. (1995), (1996), (1997)].

$$\frac{h_{max}}{h_1}=1.51F_1-0.35 \tag{4}$$

In Fig. 6, the broken line and the dotted line show the theoretical wave heights for the cnoidal wave and the solitary wave, respectively [Ippen and Harleman (1954), Iwasa (1955)].

Assuming the velocity of the water surface at the first wave crest

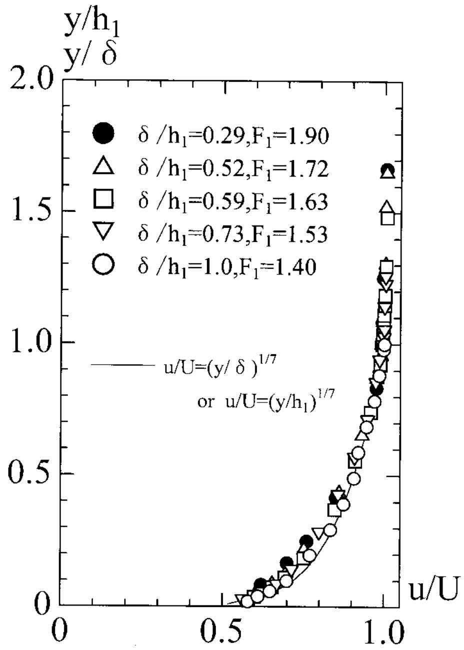


Fig. 5. Velocity distribution at toe of undular jump

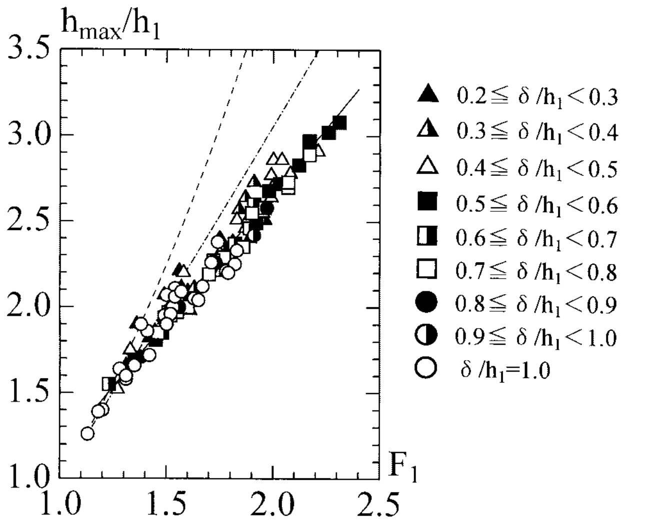


Fig. 6. Wave height of first wave crest in undular jumps

- - - - - - - - Cnoidal wave height ; $h_{max}/h_1 = 1.5(h_2/h_1) - 0.5[= 1.5\{(\sqrt{8F_1^2+1}-1)/2\} - 0.5]$

- - - - - - - Solitary wave height ; $h_{max}/h_1 = F_1^2$

———— $h_{max}/h_1 = 1.51F_1 - 0.35$

as the critical velocity [$v_c = \sqrt{gh_c}$: $v_c = q/h_c$; and $h_c = (q^2/g)^{1/3}$], and then substituting (2) into (1) yields

$$k^2\frac{F_{1limit}^{\,2}}{2} - \frac{F_{1limit}^{\,2/3}}{2} + 1 - \frac{h_{max}}{h_1} = 0 \tag{5}$$

Experimentally, for $F_1 < F_{1limit}$, the velocity of the first wave crest has a maximum at y/h_{max} = 0.6-0.8 [Fig. 7 (a)]. If the inflow Froude number approaches the upper limit of the Froude number F_{1limit}, as shown by Fig. 7 (b), the velocity of the first wave crest is at a maximum near the water surface, and this velocity becomes the critical velocity ($v_c = \sqrt{gh_c}$).

By substituting (3) and (4) into (5), the upper limit of the Froude number for undular-jump formations F_{1limit} can be predicted as (6), as shown by the solid line in Fig. 8.

$$F_{1limit} = f\left(\frac{\delta}{h_1}\right)\left[\text{i.e.,}\ \left(1-\frac{\delta}{8h_1}\right)^{-2}\frac{F_{1limit}^{\,2}}{2} - \frac{F_{1limit}^{\,2/3}}{2} + 1.35 - 1.51F_{1limit} = 0\right] \tag{6}$$

For the fully developed inflow ($\delta/h_1 = 1$), F_{1limit} = 1.786 is obtained from (6), and the value of F_{1limit} shows the same value ($F_{1limit} \fallingdotseq 1.7$) as in the case of the upper limit of the Froude number presented by Chow (1959), Bradley and Peterka (1957), and Ippen and Harleman (1954).

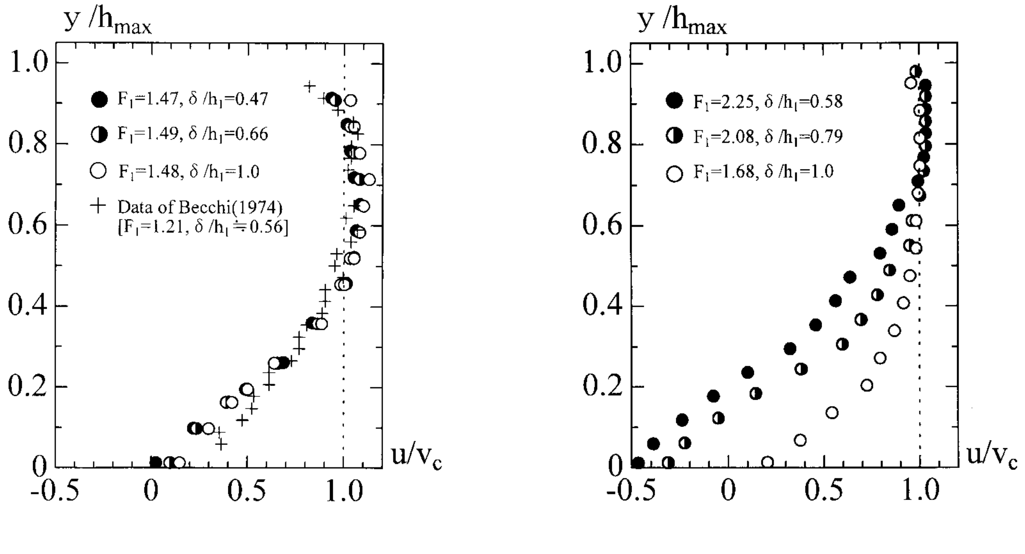
In the range of $0.45 < \delta/h_1 \leq 1.0$, as shown by Fig. 8, the predicted values agree with the experimental results. As the boundary layer thickness δ/h_1 becomes smaller, the upper limit of the Froude number becomes larger (Fig. 8), and an undular jump is formed even if the value of the inflow Froude number is larger than 1.7. For $0.18 < \delta/h_1 \leq 0.45$, as the boundary-layer thickness δ/h_1 becomes smaller, the upper limit of the Froude number becomes smaller (Fig. 8). In particular, for $\delta/h_1 < 0.22$, a jump with a surface roller is formed even if the value of the inflow Froude number is smaller than 1.7. F_{1limit} for $0.18 < \delta/h_1 \leq 0.45$ is approximated by the following equation.

$$F_{1limit} = 2.07\left(\frac{\delta}{h_1} - 0.18\right)^{\frac{1}{3}} + 1 \tag{7}$$

$$(0.18 < \delta/h_1 \leq 0.45)$$

In addition, for $\delta/h_1 \leq 0.18$, the undular jump is not formed, and a jump with a surface roller is formed. Ryabenko (1990) has also reported that a jump with a surface roller is formed immediately below a gate for any low inflow Froude number.

Fig. 9 shows the water-surface profiles of undular jumps. In this figure, x_1 is the horizontal coordinate from the toe of the jump



(a) $F_1 < F_{1limit}$

(b) $F_1 = F_{1limit}$

Fig. 7. Velocity distribution at first wave crest

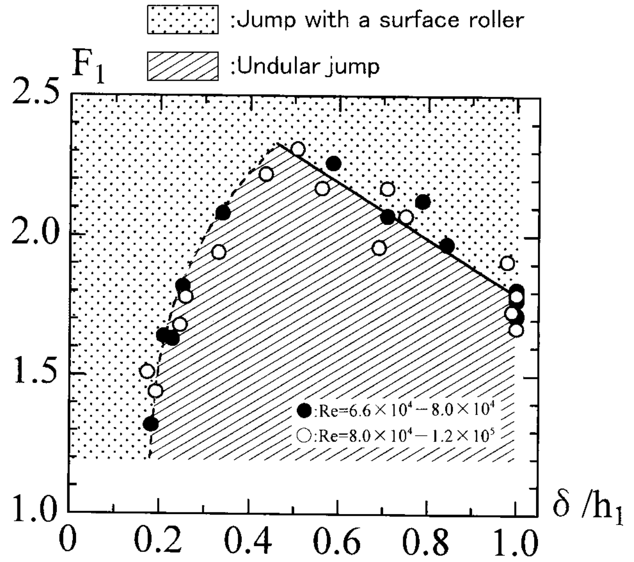


Fig. 8. Hydraulic conditions for formation of undular jumps
—— : Equation (6), ---- : Equation (7)

and h is the water depth at $x_1 = x_1$ (Fig. 1). In the range of $0.18<\delta/h_1\leq 0.45$, as the boundary-layer thickness δ/h_1 becomes smaller, the slope of the water surface becomes larger at the first and second waves, and a jump with a surface roller might be formed easily for a given inflow Froude number.

Regarding the hydraulic conditions for the formation of undular jumps below a sluice gate, the relation between the hydraulic quantities at the vena-contracta section and the toe location of the jump is significant.

The supercritical flow below a sluice gate has already been investigated, and the water-surface profile and the boundary-layer development have been predicted by using (8), (9), and (10) [Ohtsu and Yasuda (1994)].

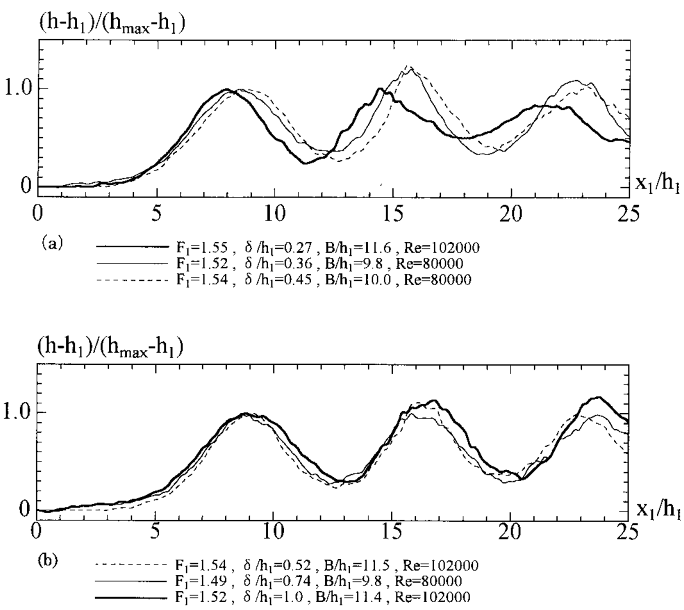


Fig. 9. Water-surface profiles of undular jumps
(a) Water surface profiles for $\delta/h_1\leq 0.45$
(b) Water-surface profiles for $0.45<\delta/h_1\leq 1.0$

$$\frac{h_1}{h_0} = \frac{1}{2}F_0^2\left(1-\frac{1}{J^2}\right)+1 \tag{8}$$

$$\frac{\delta}{h_0} = 8\left\{\frac{F_0^2}{2}\left(1-\frac{1}{J^2}\right)+1-J\right\} \tag{9}$$

$$\frac{x}{h_0} = 194Re^{\frac{1}{4}}\left\{\frac{37}{99}F_0^2\left(1-J^{-\frac{11}{5}}\right)+\frac{23}{9}(2+F_0^2)\left(J^{-\frac{1}{5}}-1\right) -\frac{8}{9}(1-J^{\frac{4}{5}})\right\}^{\frac{5}{4}} \tag{10}$$

where F_0 is the Froude number at the vena-contracta ($F_0 = U_0/\sqrt{gh_0}$), and h_0 and U_0 are the supercritical depth and the velocity at the vena-contracta, respectively, and h_1 is the supercritical depth at x = x (x = horizontal coordinate from the vena-contracta) and $J = U_0/U$ (U = velocity outside the boundary layer at x = x). In order to know the toe location of a jump at $F_1 = F_{1limit}$, the distance between the vena-contracta section and the toe location of the jump $x/h_0 = f(F_0, Re)$ has been obtained by using (6), (8), (9), (10), and $F_{1limit} = F_0(h_0/h_1)^{3/2}$ for the range of $0.45 <\delta/h_1\leq 1.0$ (the solid line in Fig. 10). For the range of $0.18 <\delta/h_1\leq 0.45$, the distance has also been obtained by using (7), (8), (9), (10), and $F_{1limit} = F_0(h_0/h_1)^{3/2}$ (the broken line in Fig. 10). As shown by Fig. 10, the hydraulic conditions required to form undular jumps below a sluice gate can be predicted. In this case, h_1 in (8) is the supercritical depth at the toe of the jump, and x in (10) is the distance between the vena-contracta section and the toe location of the jump.

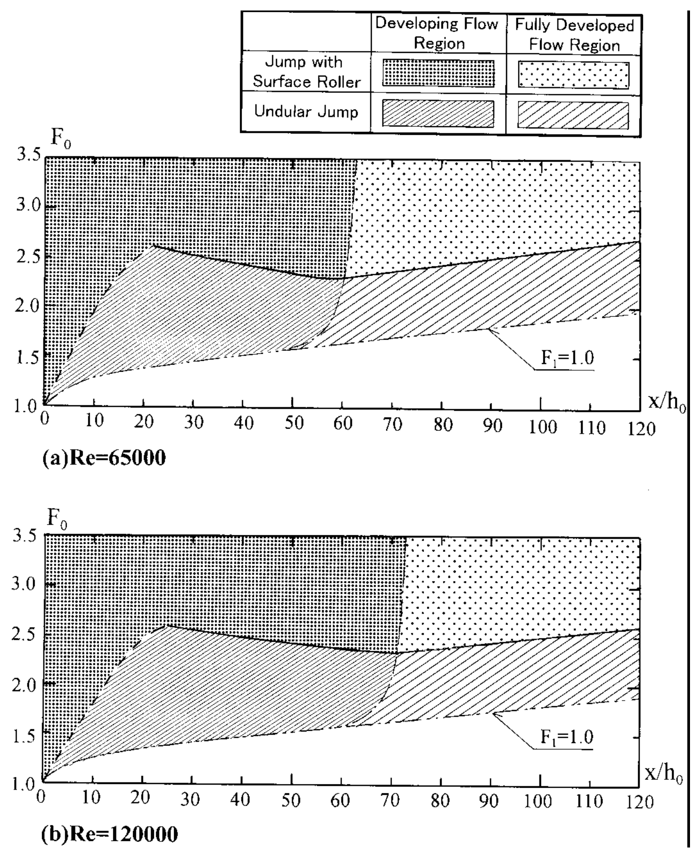


Fig. 10. Undular-jump formations below sluice gate

Comparison with studies by other researchers

Some researchers have investigated the upper limit of the Froude number for undular-jump formations in rectangular channels (see Table 2).

Ippen and Harleman (1954) proposed an upper limit of the Froude number based on theoretical considerations. Assuming an analogy between an undular surge and an undular jump, and then applying the wave theorem gave the wave height. If the wave height becomes equal to the total head at the toe of the jump, that is, the velocity at the first wave crest is assumed to be zero, the upper limit of the Froude number is obtained as $F_{1limit} = \sqrt{3}$.

Iwasa (1955), and Hager and Hutter (1984) also proposed an upper limit of the Froude number based on a theoretical approach. The theoretical wave height for open channel flows with vertical accelerations was derived, and the cnoidal wave height and the solitary wave height were shown as in the case of the wave theorem. For the upper limit of the Froude number, $F_{1limit} \fallingdotseq 1.4\text{-}1.7$ was proposed by using the theoretical wave height and assuming the velocity at the first wave crest to be zero.

As shown by Fig. 7(b), the velocity of the first wave crest near the water surface becomes the critical velocity ($v_c = \sqrt{gh_c}$) immediately before the breaking. Accordingly, their assumption might not be correct. Further, as the wave height of cnoidal wave or solitary wave is different from that of the first wave crest in undular jumps (see Fig. 6), analogy between an undular surge and an undular jump might not be applicable, except for the range of $1 < F_1 \leq 1.3$.

The upper limit of the Froude number for undular-jump formations has been proposed experimentally by many researchers [Bradley and Peterka (1957), Chanson (1995), Reinauer and Hager (1995), Binne and Orkney (1955), Ryabenko (1990)]. However, general agreement has not been obtained, because the hydraulic quantities governing the undular jumps have not been clarified. Their results can be explained by considering the effects of the boundary-layer development, the aspect ratio, and the Reynolds number on the undular-jump formations.

Conclusion

Undular hydraulic jumps in smooth rectangular horizontal channels have been investigated under conditions in which the effects of the aspect ratio and the Reynolds number on the flow characteristics are negligible.

The upper limit of the Froude number for the undular-jump formations depends on the turbulent boundary-layer development at the toe of the jump. For the developing inflow, the value of the upper limit of the Froude number shows experimentally $F_{1limit} = 1.3\text{-}2.3$. For the fully developed inflow, $F_{1limit} \fallingdotseq 1.7$ is obtained. Further, for a small boundary-layer thickness ($\delta/h_1 < 0.22$), a jump with a surface roller is formed even if the value of the inflow Froude number is smaller than 1.7.

If the inflow Froude number approaches the upper limit of the Froude number F_{1limit}, the velocity of the first wave crest is at a maximum near the water surface, and the velocity becomes the critical velocity ($v_c = \sqrt{gh_c}$) [Fig. 7(b)]. Considering the flow immediately before the breaking and the streamline along the water surface between the toe of the jump and the first wave crest gives the upper limit of the Froude number for undular-jump formations as (6). In the range of $0.45 < \delta/h_1 \leq 1.0$, the predicted values agree with the experimental results. For $0.18 < \delta/h_1 \leq 0.45$, the upper limit of the Froude number has been approximated as (7) (Fig. 8).

Further, the hydraulic condition for the formation of undular jumps below a sluice gate can be predicted by using the hydraulic quantities at the vena-contracta section and the toe location of the jump (Fig. 10).

Notations

B	channel width
F_0	Froude number at the vena-contracta
F_1	Froude number at the toe of the undular jump ($F_1 = v_1/\sqrt{gh_1}$)
F_{1limit}	upper limit of the inflow Froude number for the undular-jump formation
g	acceleration due to gravity
h	water depth at $x_1 = x_1$
h_0	supercritical depth at the vena-contracta
h_1	supercritical depth at the toe of the jump
h_2	subcritical sequent depth of the jump [$h_2 = h_1(\sqrt{(8F_1^2 + 1)}-1)/2$]
h_c	critical depth [$h_c = (q^2/g)^{1/3}$]
h_{max}	wave height of the first wave crest in the undular jump
J	$J = U_0/U$
k	correction coefficient
L	channel length
q	discharge per unit width ($q = Q/B$)
Q	discharge
R_e	Reynolds number ($R_e = v_1 h_1/\nu$)
u	mean velocity for x-direction at $y = y$
U	velocity outside the boundary layer
U_0	velocity at the vena-contracta
U_{S1}	velocity of the water surface at the toe of the jump

Table 2 Summary of experimental conditions by other researchers

Researchers	Experimental Condition								
	B(cm)	L(m)	Channel Slope i	h_1(cm)	F_1	Inflow Condition	Re	B/h_1	F_{1limit}
Bradley & Peterka(1957)	121.0,30.5	6.1	0	2.41～10.36	1.72～7.62	-----	63000～259000	3.5～40.6	1.7
Iwasa(1955)	-----	-----	-----	-----	1.29～4.14	-----	-----	-----	1.50～1.90
Binnie & Orkney(1955)	35.6	2.4	0.0055	6.4	1.12～1.52	-----	57000～77000	-----	1.26～1.55
Hager & Hutter(1986)	30	5	0.0014	-----	-----	-----	-----	-----	1.41
Ryabenko(1992)	100	39	0	3.4～21.3	1.01～2.38	Developing Inflow , Developed Inflow	31000～351000	4.7～29.4	1.58～2.00
Chanson(1995),Chanson & Montes(1995)	25	20	0.0035～0.0787	1.0～10.9	1.05～2.91	Developed Inflow	8000～120000	2.3～25	1.40～2.40
Hager & Reinauer(1995)	50	10	0	2.0～11.67	1.05～2.38	Developed Inflow	15000～145000	3.4	1.36～1.60($h_1 \geqq 6.5$～7.0cm)

[The value of Reynolds number Re were calculated by using $\nu = 1.0\times10^{-6}m^2/s$]

U_{Smax}	velocity of the water surface at the first wave crest
v_1	averaged velocity at the toe of the jump ($v_1 = Q/[Bh_1]$)
v_c	critical velocity ($v_c = \sqrt{gh_c}$)
x	horizontal coordinate from the vena-contracta
x_1	horizontal coordinate from the toe of the jump
y	vertical coordinate from channel bottom
z	transverse coordinate from the centerline of the channel
δ	boundary-layer thickness
ν	kinematic viscosity

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