Velocity profile of sediment suspensions and comparison of log-law and wake-law

Profil de vitesse des sediments en suspension et comparaison de la loi logarithmique et de la loi de sillage

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ABSTRACT
The log-law and wake-law of velocity profile of open channel flow of sediment suspensions are discussed and compared in the paper. Data from 9 literatures are employed for comparison of the two laws and regression analyses are conducted on the main factors affecting velocity profile. Empirical formulas are obtained for estimation of the factors from the flow conditions. The elevation of the maximum velocity and the deviation of velocity from the logarithmic formula at the water surface are functions of the aspect ratio of the channel. The log-law is developed into Eq. (20) applicable to the whole flow including the region near the water surface for various boundary conditions. The wake law describes the velocity distribution below the maximum velocity point. The relative error of wake-law (11%) is larger than that of log-law (6%). Moreover, the wake coefficient must be determined by using the measured velocity profile because there is no reliable formula to estimate its value from the flow conditions.

RÉSUMÉ
La loi logarithmique et la loi de sillage d'un profil de vitesse des sédiments en suspension dans un canal à surface libre sont discutées et comparées dans cet article. Les données de 9 références bibliographiques sont utilisées pour les comparaisons des deux lois st des analyses de régression sont effectuées sur les principaux facteurs affectant le profil de vitesse. Des formules empiriques sont établies pour estimer les facteurs à partir des conditions d'écoulement. La cote de la vitesse maximum et la différence avec la vitesse en surface de la loi logarithmique sont des fonctions du rapport de forme du canal. La loi logarithmique est développée dans l'équation (20) applicable à l'ensemble de l'écoulement, y compris près de la surface libre pour différentes conditions aux limites. La loi des sillage décrit la distribution de vitesse en dessous du point de vitesse maximum. L'erreur relative de la loi de sillage (11%) est plus grande que celle de la loi logarithmique (6%). De plus, le coefficient de sillage doit être déterminé en utilisant le profil de vitesse mesuré car il n'y a pas de formule fiable pour déterminer sa valeur à partir des conditions d'écoulement.

1. Introduction

The vertical velocity profile of sediment suspension in open channel has challenged scientists and hydraulic engineers for a long time. The log-law and wake-law are the most popular formulas developed from laboratory experiments. The log-law describes the profile of suspensions similar to that of clear water flow with logarithmic velocity formula but different Karman constant κ. A lot of data indicate that κ is smaller if the sediment concentration is higher (Vanoni 1946, Einstein and Chien 1955, Elata and Ippen 1961, Wang and Qian 1989).

The velocity profile of wind, slightly deviated from the log-law, appears to conform to the Coles wake function (Coles, 1956). Coleman (1981) used the wake-law to study velocity profile of sediment-laden flow in open channels and suggested that the wake coefficient rather than the Karman constant is affected by the existence of sediment. The wake-law describes the velocity profile logarithmic only in the vicinity of the bed in which the value of κ remains the same as clear water flow. It deviates from the logarithmic law in the outer region owing to the effect of the wake flow (Coleman 1981). The wake-law theory has been paid great attention and adopted by many scientists.

In both log-law and wake-law dimensional and dimensionless parameters have to be determined with measured data, U_{max} and κ for the log-law and U_{max}, κ and Π for the wake law if the profile is expressed in velocity-defect form. The wake parameter Π was suggested to be 0.55 for the air boundary layer flows (Coles, 1956). For open channel flows Coleman (1981) obtained an average Π-value of 0.19. Nezu & Rodi (1986) got Π=0~0.20, also Kirkgoz (1989) reported a value of Π=0.1. Cardoso *et al*. (1989) obtained Π-value of -0.077 over smooth bed. Kironoto and Graf (1995) found, for water flows over gravel bed, Π-value in the range from -0.08 to 0.15. It is obvious that the Π-value is not universal and may be negative and positive.

At low concentrations, the wake-law appears better than log-law as shown by Coleman. The result is not surprising because the wake law contains one more parameter determined with data. Nevertheless, the wake-law deviates far more away from the measured profile for flow of high concentrations (Wang & Larsen, 1994). Gust (1984) pointed out the overwhelming evidence for universal validity of the log law over the wake law in cohesionless flows, requiring adjustment of κ, though.

Cohesionless sediment moves mainly in the vicinity of the bed and the lower flow. Therefore, velocity profile in the inner region rather than the outer region is affected by the sediment movement, which is contrary to the theory of wake law. It is necessary

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to discuss the wake law theory again and compare the two laws for sediment-laden flows.

2. Main Parameters of Velocity Distribution

2.1 Maximum velocity

For more than one hundred years, scientists (Stearns 1883, Murphy 1904, Gibson 1909, Vanoni 1946) have found the position of the maximum velocity well below the water surface if the ratio of the width to depth, b/h, of the open channel flow is less than a value. Experiments indicate that the distance of the maximum velocity point from the bed, δ, is mainly related to the lateral position of the profile. Fig. 1 shows the relationship between δ/h and Z_0 /h of data collected from 9 literatures, in which h is the flow depth, Z_0 is the distance from the side wall. The solid curve in the figure is an empirical relationship with mathematical expression as follows:

$$\frac{\delta}{h} = 0.44 + 0.212\ \frac{Z_0}{h} + 0.05\ \sin\left(\frac{2\pi}{2.6}\frac{Z_0}{h}\right), \qquad \frac{Z_0}{h} < 2.6 \tag{1}$$

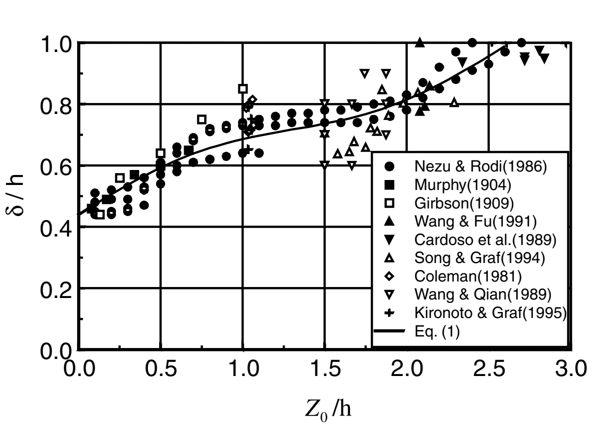
The diagram demonstrates that only if the channel is wide enough (Z_0 /h>2.6), the maximum velocity may occur at the water surface. For the central profile of an open channel flow, Z_0 =b/2, Eq.(1) is written as:

$$\frac{\delta}{h} = 0.44 + 0.106\frac{b}{h} + 0.05\ \sin\left(\frac{2\pi}{5.2}\frac{b}{h}\right), \qquad \frac{b}{h} < 5.2 \tag{2}$$

in which b is the width of the channel. If the aspect ratio b/h is smaller than 5.2, the velocity at the water surface is less than U_{max} and the position of the maximum velocity follows the formula. The data from the literatures also yield the normalized velocity difference ΔU_h^+ as a function of the width-depth ratio b/h:

$$\Delta U_h^+ = \frac{U_{hc} - U_{he}}{u_*} = 4\ e^{-0.5b/h} \tag{3}$$

in which u_* is the shear velocity, U_{he} the velocity measured at the water surface y=h . U_{hc} is defined as:



Z_0 /h

Fig. 1 Relationship of δ/h and Z_0/h

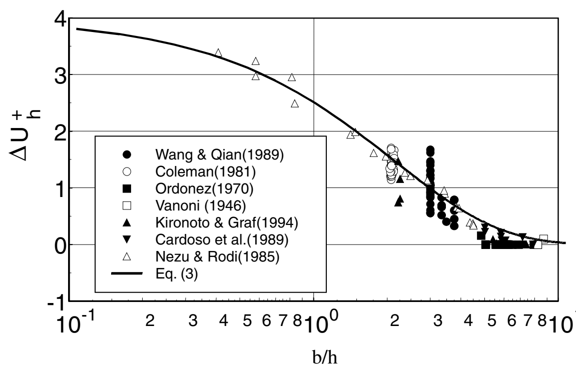


Fig. 2 The relationship of ΔU_h^+ and b/h

$$U_{hc} = U_{\max} + \frac{u_*}{0.4}\ln\left(\frac{h}{\delta}\right) \tag{4}$$

in which δ can be calculated by Eq.(2). The correlation coefficient *R* is 0.950.

In order to compare the formulas, including the velocity distribution, with measured data, the relative error E_r and correlation coefficient R are employed which are defined by:

$$E_r = \sqrt{\frac{1}{N}\sum\left(\frac{Y-\hat{Y}}{\hat{Y}}\right)^2} \tag{5}$$

$$R = \frac{\sum(X-\overline{X})(Y-\overline{Y})}{\sqrt{\sum(X-\overline{X})^2}\ \sqrt{\sum(Y-\overline{Y})^2}} \tag{6}$$

in which Y stands for the measured value and $\hat{Y}$ for the calculation with formulas, $\overline{Y}$, $\overline{X}$ are the mean values of parameters Y and X, respectively.

2.2 Effect of sediment on velocity distribution

Fig.3 shows the velocity profiles measured by Coleman (1981) and Einstein & Chien (1955), in which $C_{.05}$ (%) represents the volume concentration of sediment at y/h = 0.05. It demonstrates no obvious difference of U/u_* in the region above y/h=0.4 for

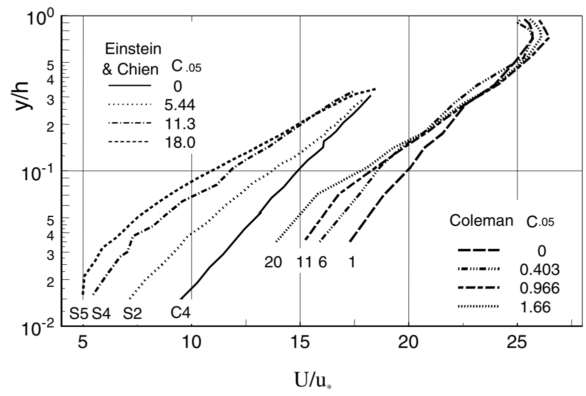


Fig. 3 Velocity profiles with different sediment concentrations

various sediment concentrations.

Fig. 4 shows the difference of flow velocity of suspension and clear water measured at y = 0.4h as a function of the reference concentration $C_{.05}$ (%). The velocity difference is small and does not increase with concentration. The average value of the difference is about zero, which interprets that the sediment does not affect the velocity in the outer region obviously.

In the vicinity of the channel bed, however, the velocity of suspensions is obviously less than that of the clear water flow (as shown in Fig. 3). Define

$$\Delta U^+_{.05} = \frac{U_{.05C} - U_{.05S}}{u_*} \tag{7}$$

to represent the velocity difference in the vicinity of bed, in which $U_{.05C}$ and $U_{.05S}$ are the velocity of clear water and suspension flow measured at y/h=0.05, respectively. $\Delta U^+_{.05}$ is found a function of sediment concentration and is related to the densities of sediment ρ_s and water ρ, Renolds number Re_t and the concentration $C_{.05}$ (%), through regression analysis expressed as follows:

$$\Delta U^+_{.05} = \frac{5700}{Re_t} \left(\frac{\rho_s - \rho}{\rho} C_{.05}\right)^{0.45} \tag{8}$$

In the formula the Reynolds number is defined as: $Re_t = R_h u_*/\nu$, here R_h is the hydraulic radius given by R_h=B x h/(2h+B) and ν the kinematic viscosity. The relationship between the velocity difference and the combined parameter of concentration, density and Reynolds number is illustrated in Fig.5. The correlation coefficient is R =0.889.

3 Velocity distribution formulas

3.1 Log-law

For smooth and rough boundary flows, the log-law velocity profiles are expressed as:

Smooth $$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(\frac{yu_*}{\nu}\right) + B_S, \quad 1 \le Re_* = \frac{K_s u_*}{\nu} \le 5 \tag{9}$$

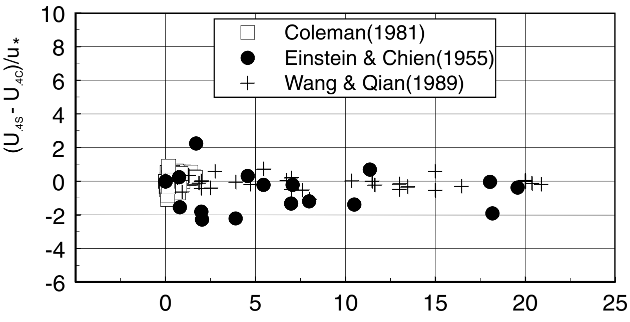
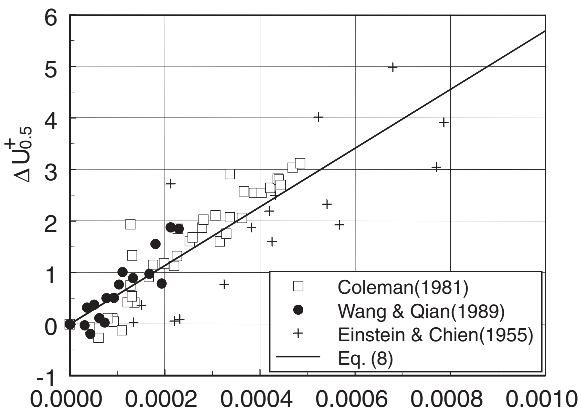


Fig.4 Velocity differences of suspension and clear water flow as a function of concentration $C_{.05}$ (%)



$$\frac{1}{R_{et}}\left(\frac{\rho_s - \rho}{\rho} C_{.05}\right)^{0.45}$$

Fig.5 Relationship between the velocity difference and the combined parameter of concentration, density and Reynolds number

Rough $$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(\frac{y}{K_s}\right) + B_f, \quad Re_* > 70 \tag{10}$$

in which U is the velocity at elevation y, κ the Karman constant, B_S and B_f the integration constants for smooth and rough boundary flows, K_s the roughness height. The flow is transitional between the two if the Reynolds number is in the region $5 < Re_* < 70$. According to Silberman *et al.* (1963), Eq.(10) can be applied to both smooth and rough boundary flows if K_s is given by:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{K_s}{14.83R_h} + \frac{2.52}{Re\sqrt{f}}\right), \quad f = 8\left(\frac{u_*}{U_m}\right)^2, \; Re = \frac{4R_h U_m}{\nu} \tag{11}$$

in which U_m represents the average velocity of the profile. Eq. (10) yields the maximum velocity U_{max} as follows:

$$\frac{U_{max}}{u_*} = \frac{1}{\kappa}\ln\left(\frac{\delta}{K_s}\right) + B_f, \quad \text{for } y <= \delta \tag{12}$$

The integration constant B_f is a function of K_s. Therefore U_{max} depends on K_s too. If U_{max} and κ are known, the velocity at elevation y can be calculated with:

$$\frac{U}{u_*} = \frac{U_{max}}{u_*} + \frac{1}{\kappa}\ln\left(\frac{y}{\delta}\right), \quad \text{for } y <= \delta \tag{13}$$

3.2 Wake-law

Coles (1956) described the wind velocity profile by combining the log-law and a wake function:

$$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(\frac{y}{K_s}\right) + B_w + \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right) \tag{14}$$

in which the constants κ and B_w are determined by using the data measured in the region y<0.15δ and the wake coefficient Π is calibrated with the data in the outer flow. By using similar method of Eqs.(12)and (13), Eq.(14) can be rewritten as:

$$\frac{U}{u_*} = \frac{U_{\max w}}{u_*} + \frac{1}{\kappa} \ln \frac{y}{\delta} + \frac{2\Pi}{\kappa} \sin^2 (\frac{\pi}{2}\frac{y}{\delta}) \quad (15)$$

The wake coefficient Π is related to the maximum velocity in the following form (Coles, 1956):

$$\Pi = \frac{\kappa}{2} (\frac{U_{\max}}{u_*} - \frac{U_{\max w}}{u_*}), \qquad \frac{U_{\max w}}{u_*} = \frac{1}{\kappa} \ln \frac{\delta}{K_s} + B_w \quad (16)$$

in which U_{max} is the measured maximum velocity in the profile. In open channel flows Π can not be directly calculated if U_{max} is unknown. For clear water flow, Nezu and Rodi (1986), Kironoto and Graf (1995) as well, obtained values of Π from experiments. Cardoso and Graf (1989) considered the wake coefficient not a universal parameter but depending on secondary currents, flow history and inactive turbulence components.

For flow of sediment suspensions, Coleman (1981) suggested the wake coefficient Π a function of Richardson number R_i:

$$\Pi = 0.2 \sqrt{R_i + 1} - 0.01 \; R_i - 0.13 \; , \quad 2 < R_i < 100 \quad (17)$$

in which:

$$R_i = \frac{g\delta\,(\rho_{.05} - \rho_\delta)}{\bar{\rho} \; u_*^2} \quad (18)$$

$\bar{\rho}$ is the average density of suspension, ρ_δ the density of suspension at y=δ (at y=0.95 δ in this paper) and $\rho_{.05}$ the density at y=0.05 δ.

3.3 Experimental data

The following experimental data (Table 1) of steady, uniform and cohesionless suspension flows are employed for discussion of the mean velocity distribution and comparison of the log-law and the wake-law.

Table 1 Experimental data collected from literatures

Authors	Number of runs	Diameter of sediment (mm)	Density of Partices (g/cm³)	Flume Wide (m)	Bed conditions	Determination of u_*
Coleman (1981)	20 12 8	0.105 0.210 0.420	2.65	0.356	Smooth	$u_* = \sqrt{gh(s_e - s_w)}$
Einstein and Chien (1955)	6 6 7	0.274 0.940 1.300	2.65	0.30	Same sand glued on bed	$u_* = \sqrt{gR_b s_e}$
Vanoni (1946)	4 1 24	0.100 0.133 0.160	2.65	0.85	0.47 , 0.88 mm sand glued on bed	$u_* = \sqrt{ghs_e}$
Wang & Qian (1989)	11 12 13	0.268 0.960 1.420	1.053	0.30	Smooth	$u_* = \sqrt{gRs_e}$
Cellino & Graf (1997)	19	0.135	2.65	0.60	Same sand covered the bed	$u_* = \sqrt{\tau_0 / \rho}$ $\tau_0 \approx (-\rho \overline{u'v'})_{y\to 0}$

(s_w is that part of the energy gradient s_e associated with head losses due to the channel walls and the channel bed away from the centerline.)

4 Verification of the log-law and wake-law

4.1 Log-law

The measured velocity deviates from the log-law in the region near the surface if the aspect ratio b/h is smaller than 5.2. The velocity difference at elevation y (y > δ) is denoted as U_d which follows the following formula:

$$\begin{aligned} \frac{U_d}{u_*} &= \Delta U_h^+ \left(\frac{y-\delta}{h-\delta} \right)^2 , \qquad for \quad y \geq \delta \\ \frac{U_d}{u_*} &= 0 \; , \qquad\qquad\qquad for \quad y < \delta \end{aligned} \quad (19)$$

in which the elevation of maximum velocity δ is given by Eq.(2). Combination of Eqs.(13) and (19) yields the velocity distribution formula for the whole profile:

$$\frac{U}{u_*} = \frac{U_{\max}}{u_*} + \frac{1}{k_p} \ln \frac{y}{\delta} - \frac{U_d}{u_*}, \quad \text{for } 0 \leq y \leq h \quad (20)$$

For flow of suspensions the Karman constant is denoted as κ_p to identify from that of clear water flow. As shown in Fig. 4 the flow velocities of clear water and suspensions at y/h = 0.4 exhibit no difference. If the log-law formula (13) or (20) is used to describe the velocity profile we may obtain

$$\frac{U_{.4S} - U_{.05S}}{u_*} = \frac{1}{\kappa_p} \ln \frac{0.4h}{0.05h} = \frac{2.08}{\kappa_p} \quad (21)$$

and

$$\frac{U_{.4C} - U_{.05C}}{u_*} = \frac{1}{\kappa} \ln \frac{0.4h}{0.05h} = \frac{2.08}{\kappa} \quad (22)$$

in which $U_{.4S}$ and $U_{.05S}$ are the velocities of suspension at y/h=0.4 and 0.05, $U_{.4C}$ and $U_{.05C}$ are the velocities of water at y/h=0.4 and 0.05. Substituting Eq (7) into Eq.(21) and comparing with Eq.(22) yields:

$$\frac{\kappa_p}{\kappa} = \frac{2.08}{\Delta U_{.05}^+ \; \kappa + 2.08} \quad (23)$$

Here κ represents the Karman constant of clear water under the same flow conditions and $\Delta U_{.05}^+$ is defined by Eq.(7) and given by Eq.(8). Fig.6 shows the relationship of κ_p/κ and $\Delta U_{.05}^+$, in which the solid line is calculated by Eq.(23) with κ =0.4 . For various experimental data the two are close with a relative error E_r = 8 %. Regression analysis further results in the maximum velocity U_{max} a function of the bed roughness and sediment concentration $C_{.05}$ (%) as shown in Fig.7. A mathematical expression of the relationship is:

$$\frac{U_{\max}}{u_*} = 13.5 - 3 \; \log \left[\frac{K_s}{h \; (1 + C_{.05})} \right] \quad (24)$$

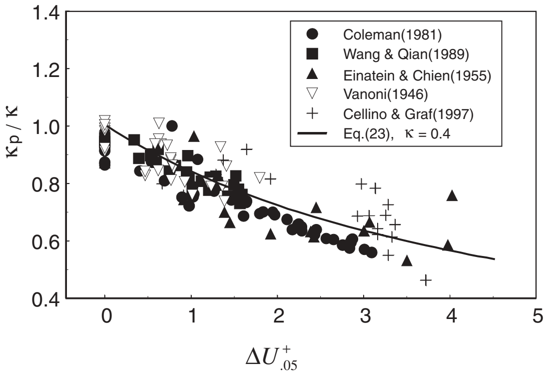


Fig. 6 The relationship of κ_p and $\Delta U^+_{.05}$

The correlation coefficient is R = -0.950.

Eqs.(20), (23) and (24) can be applied for various suspensions and channels with different aspect ratios. Fig.8 shows a comparison of Eq.(20) with data of Coleman (1981). To clearly show the velocity distribution curves they are drawn with 5 U/u_* interval. In other words, the horizontal coordinate of the second velocity profile from left is U/u_* +5, and the third is U/u_* +10 … The number shown in the diagram indicates the experiment number given by Coleman. The curves of equation (20) agree quite well with the data and the relative error is only 6%. Moreover, it is the first formula applicable to the region near the water surface ($y/\delta>1$). Because all of the parameters are calculated with universal equations (Eqs.(2), (7), (11), (19)-(24)), in other words, the parameters are estimated from flow conditions rather than measurements, the log-law is universally useful.

4.2 Wake-law

The wake-law (Eq.(15)) describes the velocity profile below the maximum velocity point ($y \le \delta$). The factors κ_w and Π are usually

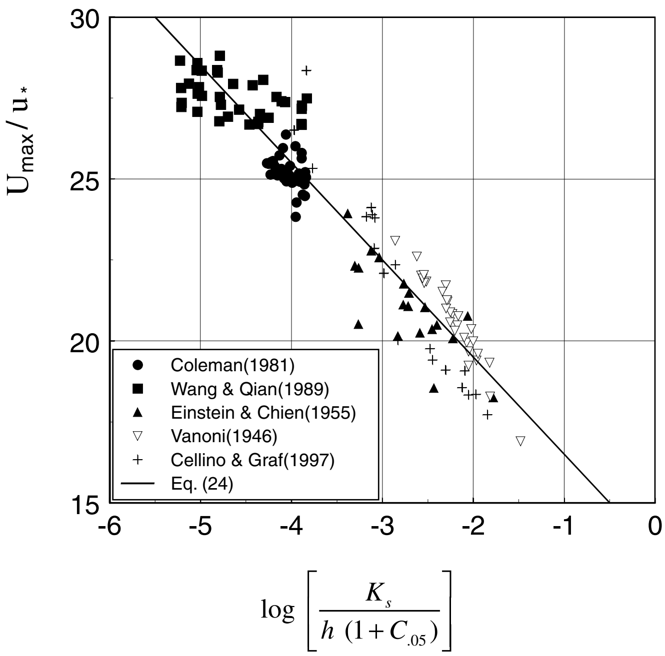


Fig. 7 The maximum velocity U_{max} as a function of K_s and $C_{.05}$

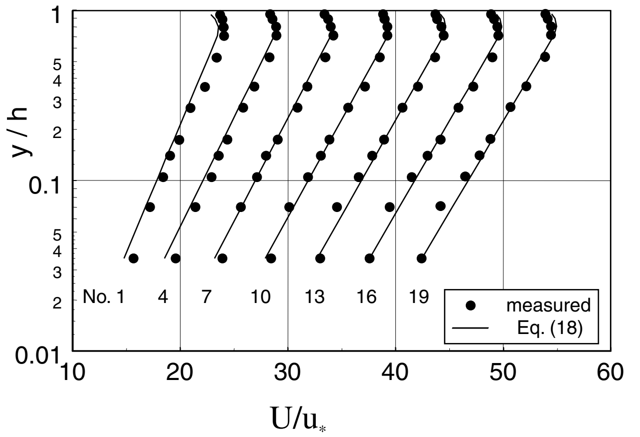


Fig. 8 Comparison of the log law and the velocity profiles by Coleman

to be determined by regression analysis of data measured in the inner region ($y \le 0.15\ \delta$). Coleman (1981) suggested an empirical formula Eq.(17) for calculation of the wake coefficient Π. Parker and Coleman (1986) suggested another formula to estimate Π latter:

$$\Pi = 0.191 + (0.4\frac{U_{m0}}{u_*}+1)\ (1-\frac{U_m C_m}{\overline{U}\ \overline{C}}+\frac{\omega}{U_{m0}J}) \quad (25)$$
$$\frac{\rho_s-\rho}{\rho}C_m - \left(\frac{U_m}{U_{m0}}-1\right)$$

The subscripts "m" and "0" stand for the average value and comparable parameters of clear water flow, J is energy slope.

The best-fit κ_w of data from the literatures is shown in Fig.9 as a function of R_i. The mean value is $\hat{\kappa}_w = 0.346$ with relative error $E_r = 229\%$. It is neither a universal constant nor equal to 0.4. Fig. 10(a) shows the relationship of best-fit Π_e of data as a function of R_i. The solid curve is Eq.(17) suggested by Coleman. The best-fit value of Π deviates from the curve with relative error $E_r = 166\%$. The wake coefficient in the diagram shows no correlation with the Richardson number. Fig.10(b) illustrates a comparison of the wake coefficient Π_e and the calculated value Π_c by Eq.(25). The two do not agree with each other and the relative error is big (E_r = 123 %).

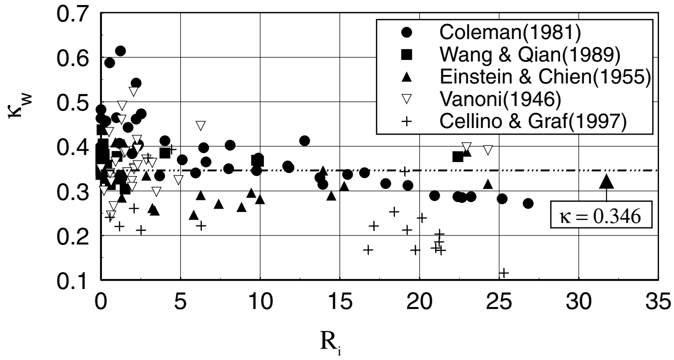


Fig. 9 The relationship of κ_w and R_i

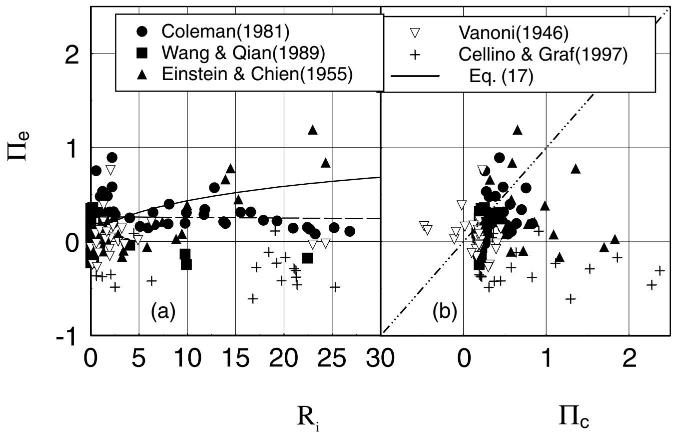


Fig. 10 Verification of wake coefficient formulas
(a) Relationship of Π_e and R_i (b) Comparison of Π_e and Π_c

Figs. 9 and 10 demonstrate no relationship between Π_e and R_i and provide no support to the formulas suggested by Coleman and Parker & Coleman. They also show the wake coefficient scattering in a big range from -0.7 to +1.2. The fact indicates that the wake function is not universal.

A comparison of Eq.(15) and measured velocity profiles by Coleman (1981) is shown in Fig. 11, in which the wake coefficient Π is calculated by using Eq(17) and U_{max} by Eq.(24). The constant κ_w is taken as 0.4 as Coleman suggested. The relative error is 11.1%, bigger than the relative error of log law (6%). The curves at y=δ is close to the data points demonstrating Eq.(24) reliable. The curves deviate from the data points in the lower part showing that the wake law is less accurate than the log-law.

Conclusions

The log-law and wake-law of velocity profile are discussed and compared. The main factors in the laws may be estimated from the flow conditions or determined with the measured velocity profiles. The elevation of the maximum velocity is a function of the aspect ratio of the channel. The log-law is developed into Eq.(20) applicable to the whole flow including the region near the water surface for various boundary conditions. The wake-law

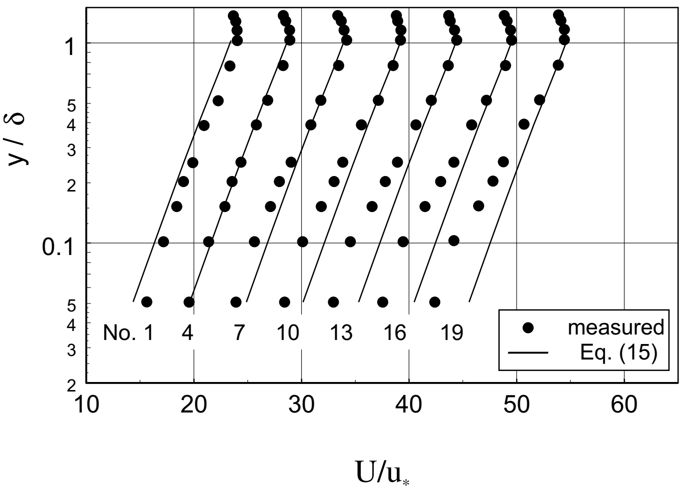


Fig. 11 Comparison of the wake law with measured velocity profiles

describes the velocity distribution below the maximum velocity point. The relative error of wake-law (11%) is larger than that of log-law (6%). Moreover, the wake coefficient must be determined by using the measured velocity profile because there is no reliable formula to estimate its value from the flow conditions.

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Notation

b = flume width;
$C_{.05}$ = sediment concentration by volume at y=0.05h;
C_{95} = sediment concentration by volume at y=0.95h;
C_m = vertical averaged concentration by volume;
D = sediment diameter;
E_r = relative error;
f = Darcy-Weisbach resistance coefficient;
h = flow depth;
K_s = roughness height;
R = correlation coefficient;
R_i = Richardson number;
U = velocity at y from the bed surface;
U_m = vertical averaged velocity;
U_{max} = maximum velocity along the vertical;
ΔU_h = difference of measured velocity near the water surface and predicted velocity by extrapolating the log-law to the same location;
$\Delta U_{.05}$ = velocity difference of suspension flow and clear water flow and $y = 0.05h$;
Z_0 = distance from the side wall;
δ = position of maximum velocity U_{max} appearing;
κ = Karman constant 0f log-law for clear water flow in the region of $y< \delta$;
κ_p = Karman constant of log-law for suspension flow in the region of $y< \delta$;
κ_w = Karman constant of wake-law for suspension flow in the region of $y<0.15\ \delta$;
Π = wake strength coefficient.