Developments in unsteady pipe flow friction modelling

Développements dans la modélisation de frottement en écoulement non permanente en conduite

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ABSTRACT

This paper reviews a number of unsteady friction models for transient pipe flow. Two distinct unsteady friction models, the Zielke and the Brunone models, are investigated in detail. The Zielke model, originally developed for transient laminar flow, has been selected to verify its effectiveness for "low Reynolds number" transient turbulent flow. The Brunone model combines local inertia and wall friction unsteadiness. This model is verified using the Vardy's analytically deduced shear decay coefficient C^* to predict the Brunone's friction coefficient k rather than use the traditional trial and error method for estimating k. The two unsteady friction models have been incorporated into the method of characteristics water hammer algorithm. Numerical results from the quasi-steady friction model and the Zielke and the Brunone unsteady friction models are compared with results of laboratory measurements for water hammer cases with laminar and low Reynolds number turbulent flows. Conclusions about the range of validity for the three friction models are drawn. In addition, the convergence and stability of these models are addressed.

RÉSUMÉ

Le papier passe en revue un certain nombre de modèles de friction non permanente en écoulement transitoire en conduite. Deux modèles de friction non permanente, celui de Zielke et celui de Brunone sont investigués en détails. Le modèle de Zielke, développé à l'origine pour les écoulements transitoires laminaires, a été sélectionné pour tester l'efficacité du modèle pour les écoulements transitoires turbulents à faible nombre de Reynolds. Le modèle de Brunone combine la variation de l'inertie locale et de la friction de paroi. Ce modèle est vérifié en utilisant le coefficient C^* d'amortissement du cisaillement de Vardy déduit analytiquement pour prédire le coefficient de friction k de Brunone, plutôt que la méthode traditionnelle par essais et erreurs pour estimer k. Les deux modèles de friction non permanente ont été incorporés dans un algorithme de calcul du coup de bélier par la méthode des caractéristiques. Les résultats numériques obtenus à partir d'un modèle de friction quasi permanente et à partir des modèles non permanents de Zielke et de Brunone sont comparés avec des résultats de mesures en laboratoire pour des écoulements laminaires ou turbulents à faible nombre de Reynolds. Des conclusions sont tirées sur les domaines de validité des trois modèles de friction. En complément, la convergence et la stabilité des modèles sont abordées.

1 Introduction

Traditionally the steady or quasi-steady friction terms are incorporated into the standard water hammer algorithms. This assumption is satisfactory for slow transients where the wall shear stress has a quasi-steady behaviour. Experimental validation of steady friction models for rapid transients [1, 2, 3, 4, 5] previously has shown significant discrepancies in attenuation and phase shift of pressure traces when the computational results are compared to the results of measurements. The discrepancies are introduced by a difference in velocity profile, turbulence and the transition from laminar to turbulent flow. The magnitude of the discrepancies is governed by flow conditions (fast or slow transients, laminar or turbulent flow) and liquid properties (viscosity). The inaccuracies in numerical model results may lead to erroneous prediction of column separation and vaporous cavitation events [5, 6, 7, 8]. Real time monitoring and control of piping systems requires accurate prediction of the pressure time history [9]. To reduce uncertainty, the influences from fluid-structure interaction and gas release should be excluded in unsteady friction validation experiments.

This paper reviews a number of unsteady friction models for transient pipe flow that have been proposed in the literature. These include an early model developed by Daily et al. [10] in which the unsteady friction is dependent on instantaneous mean flow velocity and instantaneous local acceleration. Numerous similar models have also been proposed. Brunone et al. [11] deduced an improved version in which the convective acceleration is added to Golia's [4] version of the basic Daily model. The Brunone model is relatively simple and gives a good match between the computed and measured results using an empirically predicted (by trial and error) Brunone friction coefficient k. Zielke [12] has derived a model for frequency dependent friction in transient laminar flow. The friction term is related to the instantaneous mean flow velocity and to weighted past velocity changes. The advantage of this approach is that there is no need for empirical coefficients that are calibrated for certain flow conditions. The Zielke model has been modified by several researchers to improve computational efficiency and to develop weights for transient turbulent flow. Vardy and Brown [13] deduced weighting functions from a shear layer-uniform core approximation of turbulent flow. Furthermore, the authors linked their model to the Brunone model [14] and discovered a range of validity for the instantaneous acceleration approach. The instantaneous acceleration is approximately proportional to the unsteady shear stress when the highest frequencies of interest are smaller than the reciprocal of the rise

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time. The rise time characterizes the vorticity diffusion through shear layer. Apart from one-dimensional (1D) models, which in different ways indirectly incorporate the instantaneous cross-sectional velocity profile (by empirical coefficients or weighting functions), a number of 2D models have been proposed including an early boundary layer model developed by Wood and Funk [15]. The 2D models compute an actual velocity profile and corresponding energy losses continuously during the transient process. The CPU and memory requirements for these models are large.

Two distinct unsteady friction models, the Zielke and the Brunone models, are investigated in this paper in detail. The Zielke model, originally developed for transient laminar flow, has been selected to verify its effectiveness for 'low Reynolds number' transient turbulent flow. Engineers should have better tools to be able to predict rapid transient events accurately early in the design process of the piping systems. As a result, the Brunone model is tested using Vardy's analytically deduced shear decay coefficient C^* to compute the coefficient k [14] rather than to estimate k by trial and error. The two unsteady friction models have been incorporated into the method of characteristics water hammer algorithm.

Computational results from the numerical models are compared with results of measurements performed in a laboratory apparatus comprising of 37.2 m long constant-sloping copper pipe of 22.1 mm internal diameter and a 1.63 mm wall thickness connecting two pressurized tanks. A comparison is made for the rapid closure of a downstream end valve. The performance of the Zielke and the Brunone friction models is investigated for three water hammer cases with steady state flow velocities of $V_o = \{0.10, 0.20, 0.30\}$ m/s (laminar and low Reynolds number turbulent flow). In addition, quasi-steady friction model results are also included in the paper. Conclusions about the range of validity for the unsteady friction models are drawn. Convergence and stability of the friction models are addressed.

2 Unsteady friction models

The unsteady friction terms can be classified into six groups:

1) The friction term is dependent on instantaneous mean flow velocity V (Hino et al. [16], Brekke [17], Cocchi [18]),
2) The friction term is dependent on instantaneous mean flow velocity V and instantaneous local acceleration $\partial V/\partial t$ (Daily et al. [10], Carstens and Roller [19], Safwat and van der Polder [20], Kurokawa and Morikawa [21], Shuy and Apelt [22], Golia [4], Kompare [23]),
3) The friction term is dependent on instantaneous mean flow velocity V, instantaneous local acceleration $\partial V/\partial t$ and instantaneous convective acceleration $\partial V/\partial x$ (Brunone et al. [11], Bughazem and Anderson [24]),
4) The friction term is dependent on instantaneous mean flow velocity V and diffusion $\partial^2 V/\partial x^2$ (Vennatrø [25], Svingen [26]),
5) The friction term is dependent on instantaneous mean flow velocity V and weights for past velocity changes $W(\tau)$ (Zielke [12], Trikha [27], Achard and Lespinard [28], Arlt [29], Kagawa et al. [30], Brown [31], Yigang and Jing-Chao [32], Suzuki et al. [33], Schohl [34], Vardy [35], Vardy et al. [36], Vardy and Brown [13, 14], Shuy [37], Zarzycki [38]),
6) The friction term is based on cross-sectional distribution of instantaneous flow velocity (Wood and Funk [15], Ohmi et al. [39], Bratland [40], Vardy and Hwang [41], Eichinger and Lein [42], Vennatrø [43], Silva-Araya and Chaudhry [44], Pezzinga [45]).

2.1 Unsteady friction in a standard water hammer model

The method of characteristics transformation of the unsteady pipe flow equations gives the water hammer compatibility equations which are valid along the characteristic lines [46]:

- along the C^+ characteristic line ($\Delta x/\Delta t = a$):

$$H_{i,t} - H_{i-1,t-\Delta t} + \frac{a}{g}(V_{i,t} - V_{i-1,t-\Delta t}) + \frac{f\Delta x}{2gD} V_{i,t}\left|V_{i-1,t-\Delta t}\right| = 0 \quad (1)$$

- along the C^- characteristic line ($\Delta x/\Delta t = -a$):

$$H_{i,t} - H_{i+1,t-\Delta t} - \frac{a}{g}(V_{i,t} - V_{i+1,t-\Delta t}) - \frac{f\Delta x}{2gD} V_{i,t}\left|V_{i+1,t-\Delta t}\right| = 0 \quad (2)$$

in which H = piezometric head (head), V = flow velocity, Δx = reach length, t = time, Δt = time step, a = water hammer wave speed, g = gravitational acceleration, f = Darcy-Weisbach friction factor, D = pipe diameter and i = node number. At a boundary (reservoir, valve), the boundary equation replaces one of the water hammer compatibility equations [46]. The staggered grid in applying the method of characteristics is used in this paper.

A constant value of the Darcy-Weisbach friction factor f (steady-state friction factor) is used in most of commercial software packages for water hammer analysis. As an alternative the unsteady friction factor used in Eqs. 1 and 2 can be expressed as a sum of the quasi-steady part f_q and unsteady part f_u i.e. $f = f_q + f_u$. Setting $f_u = 0$ leads to the quasi-steady friction model. The quasi-steady friction factor f_q is based on updating the Reynolds number for each new computation. For turbulent flow the Haaland explicit formula [47] is used in this paper. The Zielke and the Brunone unsteady friction models have been explicitly incorporated into the staggered grid of the method of characteristics. A first order approximation of friction term $f\Delta x/(2gD)V_{t-\Delta t}\left|V_{t-\Delta t}\right|$ is used in the two unsteady friction models.

2.2 The Zielke model

The original version of Zielke's model [12] is used in this paper. The model was analytically developed for transient laminar flow. The unsteady part of friction term is related to the weighted past velocity changes at a computational section:

$$f_{i,k} = (f_q)_{i,k} + \frac{32\nu}{DV_{i,k}\left|V_{i,k}\right|} \sum_{j=1}^{k-1} (V_{i,j+1} - V_{i,j-1}) W((k-j)\Delta t) \quad (3)$$

$$- \tau > 0.02: \quad W(\tau) = \sum_{i=1}^{5} e^{-n_i \tau} \tag{4}$$

$$- \tau \le 0.02: \quad W(\tau) = \sum_{i=1}^{6} m_i \tau^{(i-2)/2} \tag{5}$$

$$\tau = \frac{4\nu}{D^2}(k-j)\Delta t \tag{6}$$

in which j and k = multiples of the time step Δt, W = weights for past velocity changes, ν = kinematic viscosity of the fluid, τ = dimensionless time, and coefficients $\{n_i, \ i = 1, \ldots, 5\}$ = {-26.3744, -70.8493, -135.0198, -218.9216, –322.5544} and $\{m_i, i = 1, \ldots, 6\}$ = {0.282095, -1.25, 1.057855, 0.937500, 0.396696, -0.351563}.

The Zielke model requires large computer storage and has been modified by several researchers to improve the computational efficiency and/or to extend its application to the transient turbulent flow conditions [13, 14, 27 to 38]. A variable account of weights for past velocity changes is proposed in this paper to study the influence of past velocity changes for different time scales.

2.3 The original Brunone model and the Vítkovský formulation of the model

The Brunone model [11] relates unsteady friction part f_u to the instantaneous local acceleration $\partial V/\partial t$ and instantaneous convective acceleration $\partial V/\partial x$:

$$f = f_q + \frac{kD}{V|V|}\left(\frac{\partial V}{\partial t} - a\frac{\partial V}{\partial x}\right) \tag{7}$$

in which k = Brunone's friction coefficient and x = distance.

Vítkovský in 1998 investigated the original Brunone model for various flow situations. He found Eq. 7 failed to predict a correct sign of the convective term $-a\partial V/\partial x$ for particular flow and wave directions in acceleration and deceleration phases. For example, Eq. 7 fails to predict the correct sign in case of closure of the upstream end valve in a simple pipeline system with initial flow is in positive x direction. The original Brunone formulation performs correctly in case of closure of the downstream end valve [5], [8].

Vítkovský deduced a new formulation of Eq. 7:

$$f = f_q + \frac{kD}{V|V|}\left(\frac{\partial V}{\partial t} + a\,\mathrm{sign}(V)\left|\frac{\partial V}{\partial x}\right|\right) \tag{8}$$

in which sign(V) = (+1 for $V \ge 0$ or -1 for $V < 0$). Eq. 8 gives the correct sign of convective term for all possible flow and water hammer wave movement directions for either the acceleration or deceleration phases.

The Brunone friction coefficient defined for the head loss equation $h_f = fLV^2/(2gD)$ can be predicted either empirically by the trial and error method or analytically using Vardy's shear decay coefficient C^* (note: Vardy and Brown [14] are using British definition of the head loss equation $h_f = 4f_{BR}LV^2/(2gD)$ [48] for which k is 4 times larger ($f = 4f_{BR}$), C^* is identical):

$$k = \frac{\sqrt{C^*}}{2} \tag{9}$$

The Vardy's shear decay coefficient C^* from [14] is:

$$\text{- laminar flow:} \quad C^* = 0.00476 \tag{10}$$

$$\text{- turbulent flow:} \quad C^* = \frac{7.41}{\mathrm{Re}^{\log(14.3/\mathrm{Re}^{0.05})}} \tag{11}$$

in which Re = Reynolds number (Re = VD/ν).

The unknown flow velocity for time and space derivatives (Eqs. 7 and 8) in a simple finite-difference formulation (Eqs. 1 and 2) is:

$$V_{t-2\Delta t} = 0.5(V_{t-\Delta t} + V_{t-3\Delta t}) \tag{12}$$

3 Experimental apparatus

A flexible laboratory apparatus for investigating water hammer and column separation events in pipelines has been designed and constructed [49]. The apparatus comprises a straight 37.23 m (U_x = ±0.01 m) long sloping copper pipe of 22.1 mm (U_x = ±0.1 mm) internal diameter and 1.63 mm (U_x = ±0.05 mm) wall thickness connecting two pressurized tanks (Fig. 1). The uncertainty in a measurement U_x is expressed as a root-sum-square combination of bias and precision error [50]. The pipe slope is constant at 5.45 % (U_x = ±0.01 %).

A specified pressure in each of the tanks ($H_{T,1}$ and $H_{T,2}$; U_x = ±0.3 %) is controlled by a computerized pressure control system. The net water volume in both tanks and the capacity of the air compressor limit the maximum steady state velocity to 1.5 m/s and maximum operating pressure (pressure head) in each tank to 400 kPa (40 m). Water hammer events in the apparatus are initiated by rapid closure of the ball valve. Fast closure of the valve is carried out either by a torsional spring actuator (the closure time (t_c)

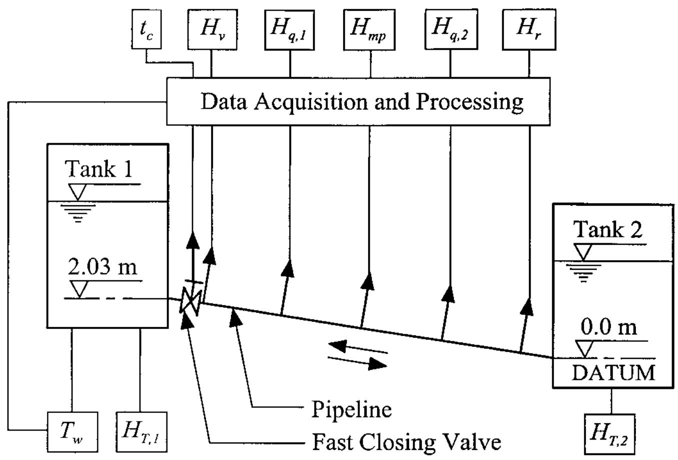

Fig. 1 Experimental apparatus layout

may be set from 5 to 10 milliseconds) or manually by hand. The actuator provides a constant and repeatable valve closure time. Five pressure transducers (H_v, $H_{q,1}$, H_{mp}, $H_{q,2}$ and H_r; $U_x = \pm0.7$ % for the piezoelectric type transducers) are located at equidistant points along the pipeline including as close as possible to the end points (Fig. 1). The water temperature in Tank 1 (T_w; $U_x = \pm0.5$ °C) is continuously monitored and the valve position during closure is measured using optical sensors ($U_x = \pm0.0001$ s for the valve closing time). Data acquisition and processing were performed using a Concurrent real-time UNIX data acquisition computer.

Each experiment using the apparatus consists of two phases. Firstly, an initial steady state velocity condition ($U_x = \pm1$ % for the volumetric method) is established. Secondly, a transient event is initiated by a rapid closure of the valve. The wave propagation velocity ($U_x = \pm0.1$ %) is obtained from the time measured for a water hammer wave to travel between the closed valve and the quarter point nearest to the valve.

4 Numerical and experimental results

Numerical and measured results of three water hammer experimental runs, with steady state flow velocities of $V_0 = \{0.10, 0.20, 0.30\}$ m/s (laminar and low Reynolds number turbulent flow), are compared to verify the performance of the Zielke and the Brunone unsteady friction models. In addition, quasi-steady friction model results are included in the analysis.

Computational and experimental runs were performed for a rapid closure of the valve positioned at the downstream end of the upward sloping pipe (Fig. 1). The flow conditions, identical for the three runs, were:

- static head in an upstream end tank $H_{T,2} = 32.$ m
- valve closure time $t_c = 0.009$ s
- water hammer wave speed $a = 1319$ m/s

The number of reaches selected for each computational run were:

- comparison analysis $N = 16$
- stability analysis $N = \{8, 16, 32\}$

Computational and experimental results for the three runs are compared at the valve (H_v) and at the midpoint (H_{mp}). The results of measurements at the two quarter points ($H_{q,1}$ and $H_{q,2}$) show similar behaviour as the results at the midpoint. The head adjacent to the Tank 2 (H_r) is the reservoir head.

4.1 Comparison of numerical and experimental results for transient laminar flow

The experimental run with flow velocity $V_0 = 0.10$ m/s is the laminar flow case with Reynolds number Re = 1870 (water temperature $T_w = 15.4$ °C). Computational results from the three friction models are compared with results of measurements and are depicted in Fig. 2.

Computational results obtained by the quasi-steady friction model

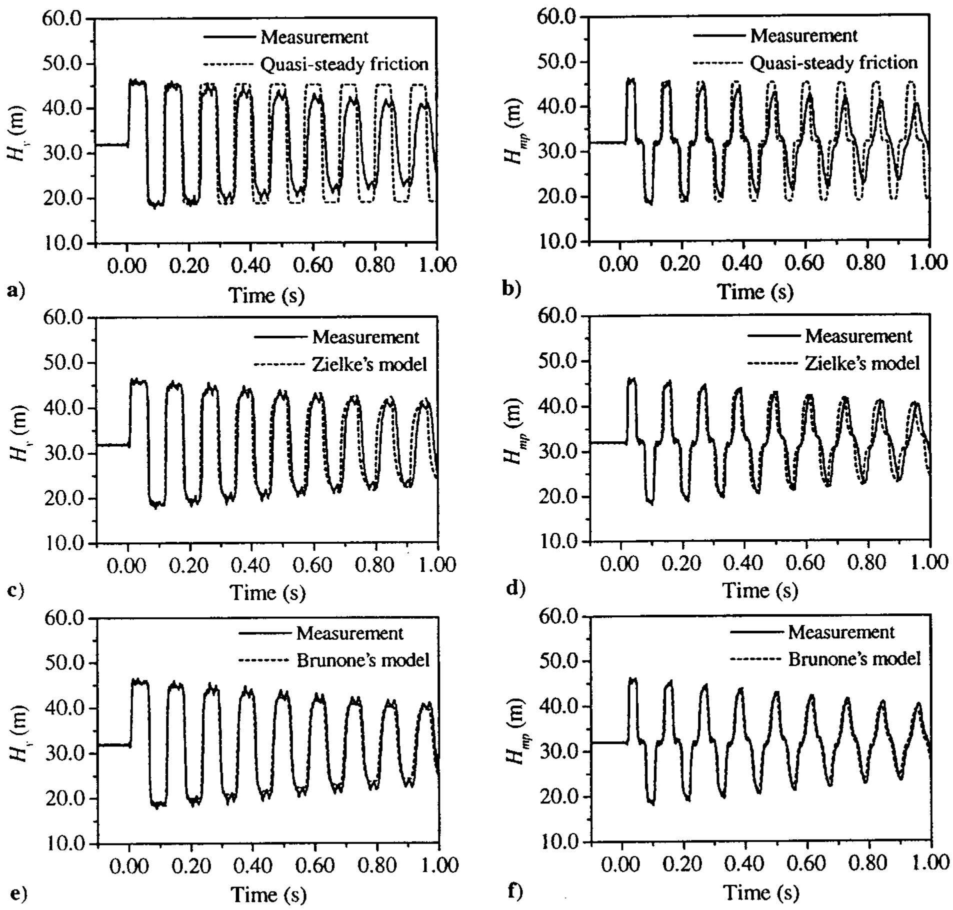

Fig. 2 Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}) in upward sloping pipe; V_0 = 0.10 m/s

(Figs. 2(a) and 2(b)) agree well for the first and the second pressure head rise. The discrepancies between the results are magnified for later times. Results from the Zielke model (Figs. 2(c) and 2(d)) show significant improvement in both attenuation and phase shift of pressure head traces. The magnitude of discrepancies in phase shift is larger than the magnitude of discrepancies in attenuation of the pressure head traces. Results from the Brunone model (Figs. 2(e) and 2(f)) exhibit the best fit. The model generates slightly excessive damping for the transient laminar flow case with analytically predicted Brunone's friction coefficient k = 0.0345 (for British definition k = 0.138). The match between the computed and measured results can be further improved by using an empirically estimated k (by trial and error) or by a more complex numerical grid (different approximation of discharges, complete method of characteristics transformation of basic equations) [5, 24]. However, the proposed explicit finite-difference scheme using k from Eq. 9 significantly improves the results and shows a significant potential for improving the Brunone model.

The ratio of CPU user time for the quasi-steady:Zielke:Brunone model is 1:45:1.2. The CPU user time for Zielke's model increases rapidly for a larger number of reaches. Decreasing the number of past velocity changes used in Zielke's analysis can increase the computational efficiency of the model. This provides a negligible loss of accuracy. For example, a $4L/a$ storage time decreases the CPU user time by a factor of three.

4.2 Comparison of numerical and experimental results for transient turbulent flow

Two experimental runs with initial flow velocities V_0 = {0.20, 0.30} m/s represent low Reynolds number turbulent flows. The corresponding Reynolds numbers are Re = {3750, 5600 at water temperature T_w = {15.4, 15.5} °C}. The results from the three friction models are compared with measured results and are shown in Figs. 3 and 4 respectively.

The quasi-steady state friction results for the two velocity cases (Figs. 3(a) and 3(b) for V_0 = 0.20 m/s; Figs. 4(a) and 4(b) for V_0 = 0.30 m/s) show similar behaviour compared to the results for the laminar flow velocity case (Figs. 2(a) and 2(b) for V_0 = 0.10 m/s). Results from the Zielke model (Figs. 3(c) and 3(d) for V_0 = 0.20 m/s; Figs. 4(c) and 4(d) for V_0 = 0.3 m/s) show significant improvement both in attenuation and phase shift of pressure head traces when compared to the quasi-steady model results. Inclusion of weights for past velocity changes developed for transient laminar flow increase the rate of energy loss for low Reynolds number transient turbulent flow (Re < 10^4) [5, 27]. The magnitude of discrepancies between computed and measured results for low Reynolds number turbulent flow (Figs. 3(c) and 3(d) for V_0 = 0.20 m/s; Figs. 4(c) and 4(d) for V_0 = 0.30 m/s) is similar to that for laminar flow (Figs. 2(c) and 2(d) for V_0 = 0.10 m/s). The results of the calculation can be further improved by inclusion of weighting functions for transient turbulent flow [13, 14]. Results

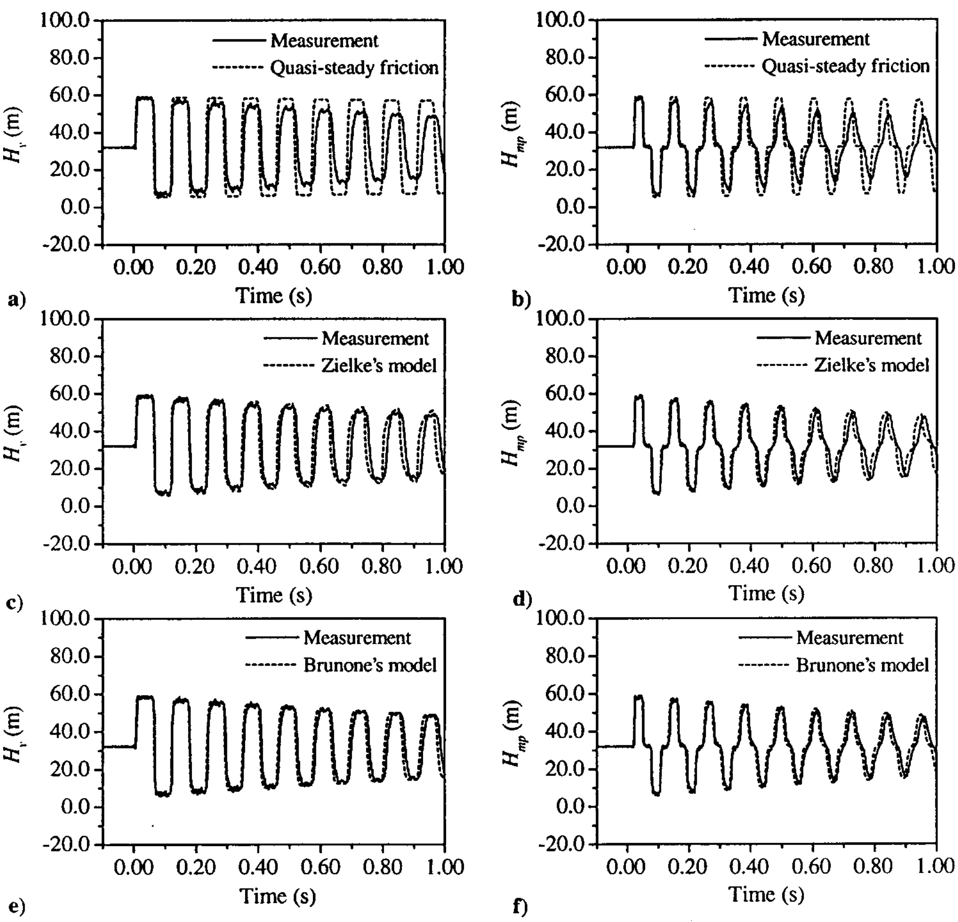

Fig. 3 Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}) in upward sloping pipe; V_0 = 0.20 m/s

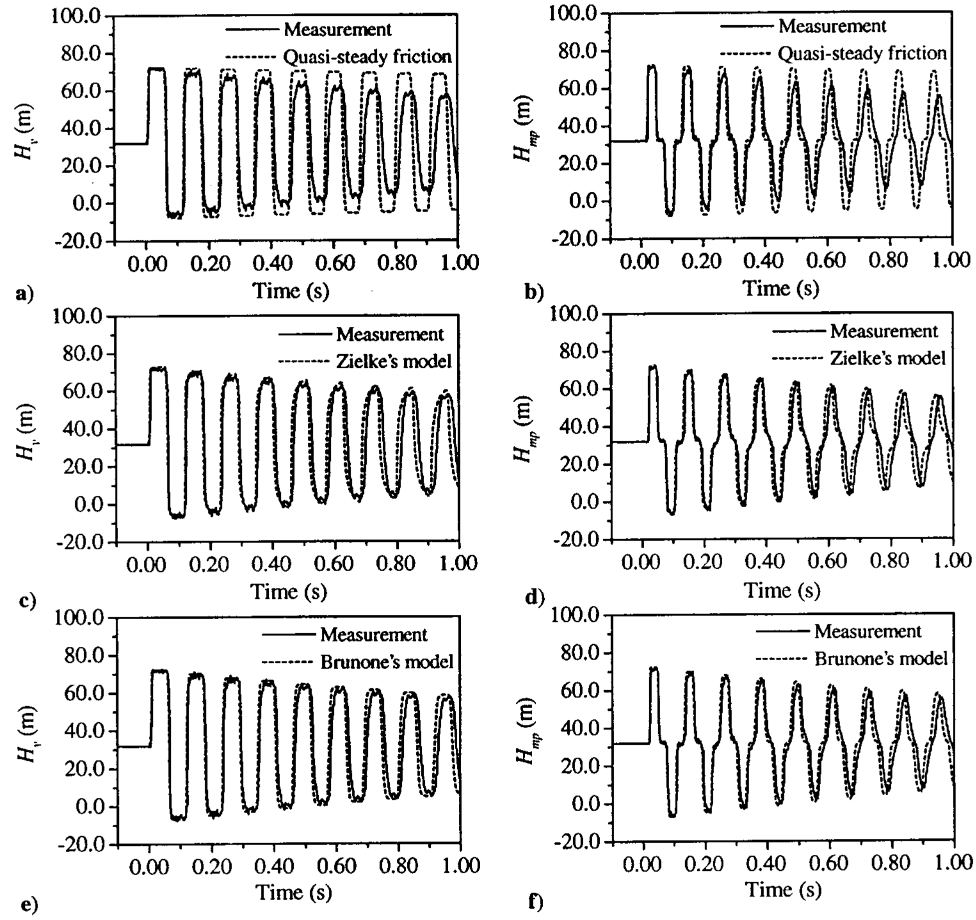

Fig. 4 Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}) in upward sloping pipe; V_0 = 0.30 m/s

from the Brunone model (Figs. 3(e) and 3(f) for V_0 = 0.20 m/s; Figs. 4(e) and 4(f) for V_0 = 0.30 m/s) show the best match with experimental results. The model generates slightly lesser attenuation with a minor phase shift of pressure pulses. The analytically predicted Brunone's friction coefficient (Eq. 9) for velocity V_0 = 0.20 m/s is k = 0.0245 (for British definition k = 0.098) and for velocity V_0 = 0.30 m/s is k = 0.0209 (for British definition k = 0.0836). The performance of the Brunone model, as for the case of transient laminar flow, can be improved by empirically estimating k or by using a more complex numerical grid [5, 24]. The comparison clearly shows the dependence of k on the Reynolds number [14]. The next step in research is to incorporate a variable $k = k(\mathrm{Re})$ into the Brunone model. A similar comparison analysis for a broad range of high Reynolds number turbulent flows (10^5 to 10^7) would be highly desirable. Construction of a new experimental apparatus for investigating high Reynolds number turbulent flows is planned in ongoing research.

4.3 Convergence and stability

The numerical solution of unsteady friction models incorporated into the method of characteristics computational grid should satisfy convergence and stability criteria. Convergence relates to behaviour of the solution as Δx and Δt tend to zero while stability is concerned with round-off error growth [51]. The performance of unsteady friction models is examined for a number different Δx and Δt sizes to determine each model's internal consistency.

Bughazem and Anderson [24] reported a numerically unstable solution using the explicit finite-difference form of the Brunone model on the rectangular grid of the method of characteristics. The solution was strongly dependent on the number of reaches (effecting Δx and Δt). A method of characteristics implementation of the Brunone model using a space-line representation performed consistently for a broad range of numbers of reaches N (4 to 20). No inconsistencies using quasi-steady and Zielke models have been reported in the literature.

The influence of different number of reaches (N = 8, 16 and 32) is investigated for the three friction models presented in this paper. Examination of computational results reveals numerically stable behaviour of all three models. Fig. 5 shows that the results for the Brunone model using the staggered grid of the method of characteristics are consistent for various numbers of reaches for the three velocity cases that are presented in comparison analysis.

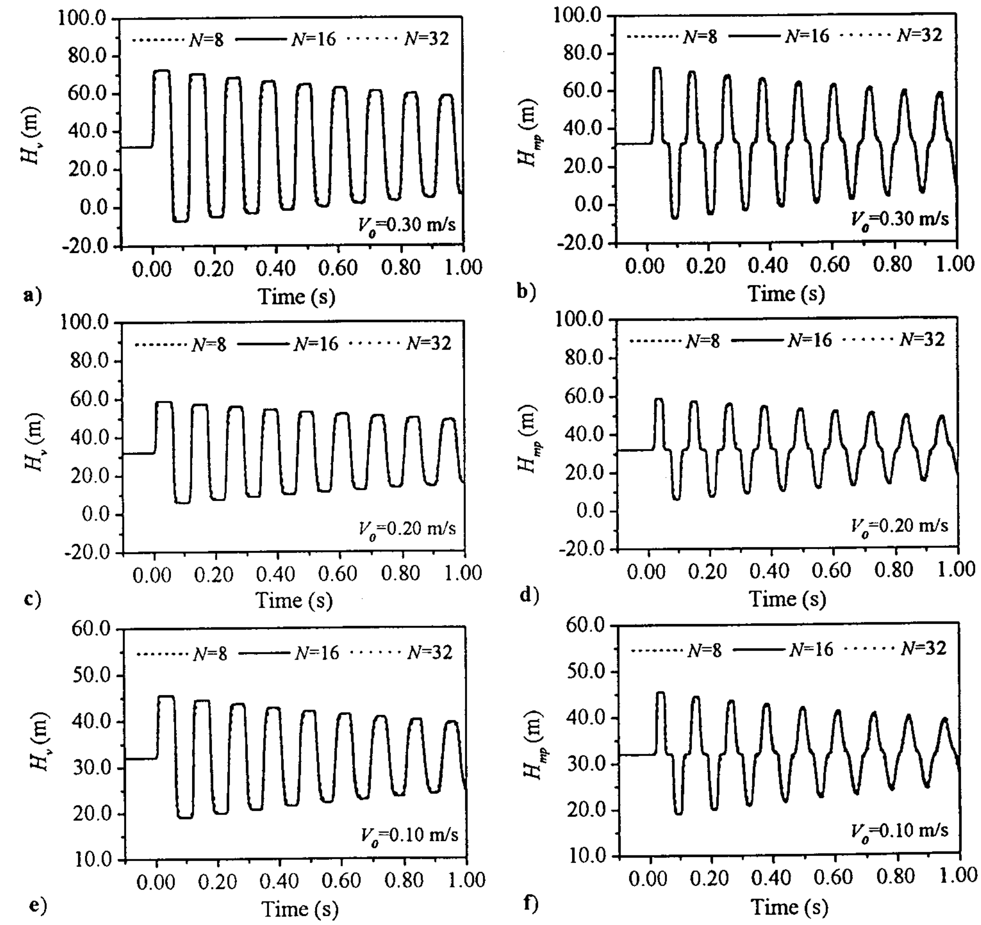

Fig. 5 Numerical analysis of Brunone's model

Conclusions

Results from the quasi-steady friction model, the Zielke model and the Brunone model have been compared with results of measurements from a fast valve closure in a laboratory apparatus. A comparison analysis includes experimental runs for laminar flow and low Reynolds number turbulent flows ($\mathrm{Re} < 10^4$). The Zielke model, developed originally for laminar flow, performs effectively for low Reynolds number turbulent flow. The best fit is given by the Brunone model even when using explicit finite-difference scheme and an analytically predicted Brunone's friction coefficient k which was calculated from Vardy's shear decay coefficient C^*. These results clearly indicate the dependence of k on the Reynolds number. The influence of different number of reaches was also investigated. Examination of computational results reveals numerically stable behaviour for all three friction models.

Notation

a	water hammer wave speed
C^*	Vardy's shear decay coefficient
D	pipe diameter
f	Darcy-Weisbach friction factor
g	gravitational acceleration
h_f	friction-head loss
H	piezometric head (head)
k	Brunone's friction coefficient
L	pipe length
m_i	coefficient in the Zielke model
N	number of computational reaches
n_i	coefficient in the Zielke model
Re	Reynolds number = VD/ν
T_w	water temperature
t	time
t_c	valve closure time
U_x	uncertainty in a measurement
V	flow velocity
W	weighting function
x	distance
Δt	time step
Δx	reach length
ν	kinematic viscosity
τ	dimensionless time

Subscripts

i	node number
j	multiple of the time step Δt
k	multiple of the time step Δt
mp	midpoint
q	quasi-steady part
r	reservoir

T pressurized tank
u unsteady part
v valve
0 steady state

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