Numerical simulation and prevention of water freezing in outdoor penstocks

Simulation numérique et prévention du gel de l'eau dans les conduites extérieures sous pression

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ABSTRACT

Pipes laid in open air and conveying water can freeze in winter times. A blockage due to freezing can be effectively prevented by providing a minimum discharge. This precaution ensures the free flow of water through the pipe, but does not stop the forming of an ice layer on the inner pipe wall. In this paper a mathematical model is developed to determine minimal protection discharge and simulation of variation in time along the pipe of ice layer formed inside outdoor pipes during non-stationary atmospheric regime. The model allows the study of the pipe capacity to transport the normal discharge in operation and minimal protection discharge without affecting the hydraulic characteristics of the flow. Also, it gives the possibility to adopt economical solutions for the problem of protecting these pipes from frost. The performance of the developed model is illustrated using a numerical example.

RÉSUMÉ

Les conduites situées à l'extérieur et l'eau que celles-ci transportent peuvent geler pendant l'hiver. Le blocage dû au gel peut être prévenu efficacement en assurant un débit minimal. Cette précaution assure l'écoulement libre de l'eau dans la conduite, mais ne stoppe pas la formation d'une couche de glace sur la paroi intérieure. L'article développe un modèle mathématique pour déterminer le débit minimum de protection et pour simuler l'évolution, dans le temps et le long de la conduite, du gel de l'eau à l'intérieur des conduites extérieures en charge, en régime atmosphérique non-stationnaire. Cela permet d'étudier si la conduite est capable de transporter le débit normal d'exploitation, ainsi que le débit minimum de protection, sans affecter les caractéristiques hydrauliques de l'écoulement. Aussi offre-t-il la possibilité d'adopter des solutions économiques de protection de ces conduites contre le gel. La performance du model développé est illustrée par un exemple numérique.

1. Introduction

Using outdoor pipes in hydroelectric plants and water supply systems (penstocks, valley and river crossings [6] etc.) during cold periods, severe difficulties can occur. They are due, either to water freezing which may lead to the formation of an ice layer on the inner pipe wall, affecting the flow due to hydraulic characteristics desired, or to permanent deformations induced by this phenomenon.

Therefore, if the water flow is interrupted in order to perform various maintenance operations in the system, or according to the working program of the hydroelectric power station, and the pipe is full of water, after some time the whole water mass in the pipe will freeze. In order to prevent this phenomenon, in practice there is a tendency to ensure a permanent minimum discharge through the pipe. For resting atmospheric regime (wind velocity negligible), a relation to calculate the minimal discharge to prevent water freezing [7] was determined.

This precaution, in most cases, ensures free flow in the pipe, but does not stop the formation of an ice layer on the pipe inner wall, as for a water temperature near 0 °C it is impossible to avoid water freezing, even for flow velocities over 10 m/s.

Therefore, we must study if the pipe is able to transport the minimal protection discharge for a non-stationary atmospheric regime, without affecting the hydraulic characteristics of the flow. Also, even in the case of a normal operation with a given discharge, it is necessary to study the capacity of the pipe to transport the discharge, considering the ice volume that can be formed inside. If the pipe is not adequate, it must be protected through outside protective thermal covers.

This paper offers theory on ice layer formation in outdoor penstocks in regions of frost endangered climate, and bring numerical examples. The paper presents a mathematical model for simulation of changes in time along the pipe of ice layer formed inside outdoor pressurized pipes, under non-stationary atmospheric regime. This model can be used for obtaining economical solutions of the problem to protect these pipes from frost.

2. Statement of the problem and theoretical approach

2.1 Elements of thermal energy

In figure 1 are presented the longitudinal section of a frozen pipe with elementary length *dx*, and its characteristic thermal features. The direction of *x*-coordinate coincides with flow direction.

By examining the thermal phenomena inside the pipe, we conclude that the main terms of thermal energy to be considered in calculating the thermal balance for a unit length pipe, are:

– Heat flow water, Q_w [J/day]:

$$Q_w = 86400 G \rho_w c_w t_w \tag{1}$$

in which: G is the discharge through the pipe; ρ_w – water density; c_w – specific heat of water; t_w – cross-section averaged water temperature.

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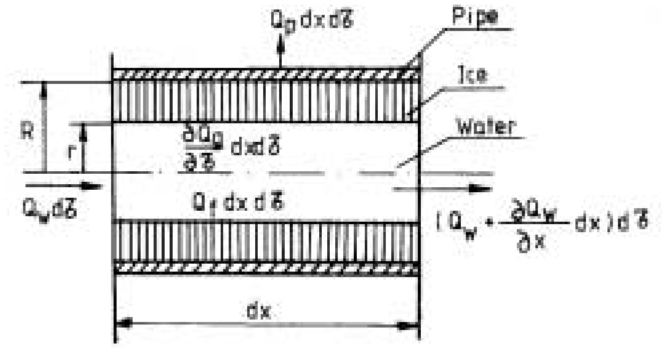

Fig. 1. Longitudinal section through a pipe with frozen layer

– Heat developed by friction, Q_f [J/(m day)]:

$$Q_f = 86400\rho_w gGJ = \frac{54600\pi\rho_w g}{n} r^{2.67} J^{1.5} \tag{2}$$

in which: g is the gravitational acceleration; J – hydraulic slope; n – Manning roughness coefficient; r – radius of inside cross-section available to discharge.

– Heat transmitted by water to external air through pipe wall, Q_p [J/(m day)]:

$$Q_p = \frac{86400 \times 2\pi(t_1 - t_{N+1})}{\frac{1}{\alpha_i r_1} + \sum_{j=1}^{N-1} \frac{1}{\lambda_j} \ln\frac{r_{j+1}}{r_j} + \frac{1}{\alpha_e r_N}} \tag{3}$$

in which: α_i, α_e are the coefficients of internal and external convection; λ_j – thermal conductivity of material layer j of the pipe; N – number of material layers.

The thermal resistance of convective heat transfer from water to pipe (α_i has high values) and the thermal resistance of the pipe wall, can be neglected due to their reduced influence on the thermal flow:

$$Q_p = \frac{86400 \times 2\pi(t_1 - t_{N+1})}{\frac{1}{\lambda_g} \ln\frac{R}{r} + \frac{1}{\alpha_e R}} \tag{4}$$

in which λ_g is the thermal conductivity of ice; R – inner radius of pipe.

Considering the expression for the coefficient of external convection [5]:

$$\alpha_e = 3.77\frac{w^{0.7}}{R^{0.3}}, \tag{5}$$

w being the wind velocity, and using the substitution:

$$\frac{1}{\alpha_e R} = -\frac{1}{\lambda_g} \ln\frac{R}{R_f} \tag{6}$$

we can determine a fictitious radius R_f, which considers the disturbances of the external air flow:

$$R_f = R \cdot \mathrm{e}^{0.615/(wR)^{0.7}} \tag{7}$$

Introducing the relations for temperature $t_1 = t_g = 0$ °C and $t_{N+1} = t_e$, the formula (4) takes the form:

$$Q_p = \frac{-86400\pi t_e}{-\frac{1}{2\lambda_g} \ln\frac{r}{R_f}} \tag{8}$$

in which: t_e is the external air temperature; t_g – ice melting temperature.

– Solidification heat of water, Q_g [J/m]:

$$Q_g = \pi S_g(R^2 - r^2)L_w \tag{9}$$

in which: ρ_g is the ice density; L_w – specific solidification heat of water.

– Heat transmitted by water to ice, Q_t [J/(m day)]:

$$Q_t = 86400 \text{ x } 2\pi r\alpha_i(t_w - t_g) \tag{10}$$

The coefficient of internal convection can be determined with the relation:

$$\alpha_i = 416\frac{G^{0.75}}{r^{1.75}} \tag{11}$$

Ice melting temperature varies with water pressure head in the pipe [1]:

$$t_g = -0.784 \text{ x } 10^{-3}H \tag{12}$$

in which H is the water pressure head.

Introducing (11) and (12) into (10) the following expression is obtained:

$$Q_t = 72\times10^6\pi(t_w + 0.784\times10^{-3}H)\left(\frac{G}{r}\right)^{0.75} \tag{13}$$

2.2 Thermal balance

The thermal balance of the water through the pipe – ice – external environment system can be expressed as:

$$-\frac{\partial Q_w}{\partial x} + Q_f - Q_p + \frac{\partial Q_g}{\partial \tau} = 0 \tag{14}$$

in which τ is the time.

For the ice – pipe system, the thermal balance can be written as:

$$Q_t - Q_p + \frac{\partial Q_g}{\partial \tau} = 0 \tag{15}$$

The thermal balance of water flow through the pipe is represented as follows:

$$-\frac{\partial Q_w}{\partial x}+Q_f-Q_t=0 \tag{16}$$

2.3 Mathematical/Numerical model

Due to water temperature and pressure variation in time along the pipe, the ice layer depth will vary as well. Therefore, generally, $r = r(x,\tau)$. In the numerical simulation, the following values of parameters are used: $\rho_w = 1000$ kg/m^3, $g = 9.81$ m/s^2, $c_w = 4185$ J/(kg K), $L_w = 333.3\times10^3$ J/kg, $\rho_g = 917$ kg/m^3, $\lambda_g = 2.32$ W/(m K), $n = 0.01$.

2.3.1 Constant ice layer depth along the pipe
For pipes with large length ($x \rightarrow \infty$) one may assume a cylindrical ice cover formed on the inner pipe wall:

$$\frac{\partial r}{\partial x}=0;\quad \frac{\partial r}{\partial \tau}=\frac{dr}{d\tau}, \tag{17}$$

Defining the relative radius of ice (non-dimensional parameter):

$$r_*=\frac{r}{R_f} \tag{18}$$

and considering the hydraulic slope J, according to the Chézy-Manning formula:

$$J=0.255\frac{n^2G^2}{R_f^{5.33}}\frac{1}{r_*^{5.33}} \tag{19}$$

as well as relations (1), (2), (8) and (9), the equation (14) takes the form:

$$\begin{aligned}&-86400G\rho_w c_w\frac{\partial t_w}{\partial x}+22000\frac{\rho_w g n^2G^3}{R_f^{5.33}}\frac{1}{r_*^{5.33}}-\\&\frac{86400\pi t_e}{\frac{1}{2\lambda_g}\ln r_*}-2\pi\rho_g L_w R_f^2 r_*\frac{dr_*}{d\tau}=0\end{aligned} \tag{20}$$

Using the relations (8), (9) and (13), the equation (15) takes the form:

$$\begin{aligned}&72\times10^6\pi(t_w+0.784\times10^{-3}H)\left(\frac{G}{R_f}\right)^{0.75}\frac{1}{r_*^{0.75}}-\\&\frac{86400\pi t_e}{\frac{1}{2\lambda_g}\ln r_*}-2\pi\rho_g L_w R_f^2 r_*\frac{dr_*}{d\tau}=0\end{aligned} \tag{21}$$

Considering pressure head gradient:

$$\frac{\partial H}{\partial x}=J_p-J=J_p-0.255\frac{n^2G^2}{R_f^{5.33}}\frac{1}{r_*^{5.33}}, \tag{22}$$

the expression of the parameter t_w is determined from the equation (21) and is derived with respect to x as follows:

$$\frac{\partial t_w}{\partial x}=-0.784\times10^{-3}\left(J_p-0.255\frac{n^2G^2}{R_f^{5.33}}\frac{1}{r_*^{5.33}}\right), \tag{23}$$

in which J_p is the pipe slope.
Substituting the relation (23) and the values of known parameters into (20), the following differential equation is obtained:

$$\begin{aligned}\frac{dr_*}{d\tau}=&\,0.148\frac{J_pG}{R_f^2}\frac{1}{r_*}+7.48\times10^{-6}\frac{G^3}{R_f^{7.33}}\frac{1}{r_*^{6.33}}-\\&0.66\times10^{-3}\frac{t_e}{R_f^2}\frac{1}{r_*\ln r_*},\end{aligned} \tag{24}$$

which allows the study of ice layer depth variation in time.
As water temperature is, generally, known in practice, it can be expressed by introducing expression (24) in equation (21):

$$\begin{aligned}t_w=&\,1.256J_pG^{0.25}R_f^{0.75}r_*^{0.75}+\\&0.636\times10^{-4}\frac{G^{2.25}}{R_f^{4.58}}\frac{1}{r_*^{4.58}}-0.784\times10^{-3}H\end{aligned} \tag{25}$$

The derivative $dr_*/d\tau$ from equation (24), under certain given conditions, varies only vs the relative radius r_*. The graph of this function (fig. 2) intersects the axis of coordinates ($dr_*/d\tau=0$) for the following value:

$$r_*=r_{*lim} \tag{26}$$

which represents the relative limiting radius towards which the

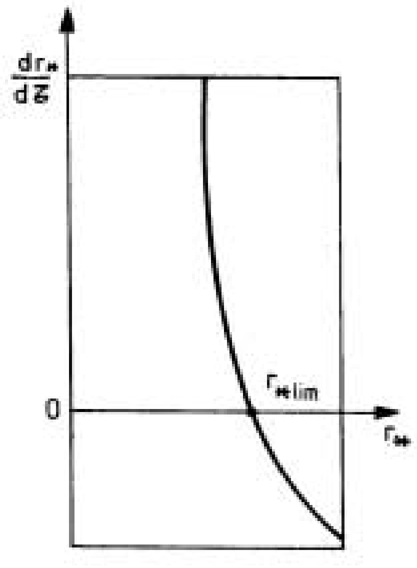

Fig. 2. Graph of the function $dr_*/d\tau$

ice surface in the pipe tends, under the given conditions.

By the forming of ice on the pipe wall, the roughness coefficient decreases, so that for a certain thickness of ice, characterized by the maximal admitted relative radius:

$$r_{*ad} = 0.705\frac{R}{R_f} \tag{27}$$

the transportation capacity can be maintained without endangering the system by strongly increasing water velocity through the pipe.

Protection against frost is leading to a completely free-of-ice pipe can result in extremely high investment costs for insulation. Therefore, if for certain given (specified) conditions it results that:

$$r_{*lim} \geq r_{*ad} \tag{28}$$

then, the pipe can function without thermal insulation. If condition (28) is not satisfied, it is recommended to determine the thickness of thermal insulation layers accepting the formation of an ice layer characterized by the relative radius equal to r_{*ad}. In this case, we must also use relation (3).

For $G = 0$ in expression (24), and integrating the resulting equation with the boundary condition:

$r_* = R/R_f$, for $\tau = 0$, and $r_* = r_{*ad}$, after introducing expression (27), we obtain the relation of maximum admissible time for water stagnation in the pipe, τ_{max} [days]:

$$\tau_{max} = 758\frac{R^2}{t_e}\left(0.503\ln\frac{R}{R_f} - 0.076\right) \tag{29}$$

2.3.2 Variable ice layer depth along the pipe

For pipes operating at constant hydraulic and thermal characteristics for a long time ($\tau \rightarrow \infty$) the assumption of the forming of an ice layer of variabled depth along the pipe is considered:

$$\frac{\partial r}{\partial \tau} = 0; \quad \frac{\partial r}{\partial x} = \frac{dr}{dx}, \tag{30}$$

In this case, the expressions (20) and (21) of thermal balance equations (14) and (15) take the forms:

$$-86400G\rho_w c_w\frac{\partial t_w}{\partial x} + 22000\frac{\rho_w g n^2 G^3}{R_f^{5.33}}\frac{1}{r_*^{5.33}} - \frac{86400\pi t_e}{\frac{1}{2\lambda_g}\ln r_*} = 0 \tag{31}$$

$$72\times10^6\pi(t_w + 0.784\times10^{-3}H)\left(\frac{G}{R_f}\right)^{0.75}\frac{1}{r_*^{0.75}} - \frac{86400\pi t_e}{\frac{1}{2\lambda_g}\ln r_*} = 0 \tag{32}$$

The expression of the parameter t_w is determined by the equation (32):

$$t_w = 0.00559t_e\left(\frac{R_f}{G}\right)^{0.75}\frac{r_*^{0.75}}{\ln r_*} - 0.784\times10^{-3}H \tag{33}$$

where:

$$H = H_o + \left(J_p - 0.255\frac{n^2G^2}{R_f^{5.33}}\frac{1}{r_*^{5.33}}\right)x \tag{34}$$

in which H_o is the hydraulic head available of the pipe.

Deriving the expression (33) with respect to x and introducing in (31) the known values, we obtain the differential equation:

$$\frac{dr_*}{dx} = -0.00714\times10^{-3}\frac{G^{2.75}}{t_e R_f^{6.08}}\frac{(-\ln r_*)^2}{r_*^{5.08}\left[1 + 0.75(-\ln r_*)\right]} - \\ -0.625\times10^{-3}\frac{1}{G^{0.25}R_f^{0.75}}\frac{r_*^{0.25}(-\ln r_*)}{1 + 0.75(-\ln r_*)} \\ -0.1407\frac{J_p G^{0.75}}{t_e R_f^{0.75}}\frac{r_*^{0.25}(-\ln r_*)^2}{1 + 0.75(-\ln r_*)} \tag{35}$$

which describes ice layer variation along the pipe.

The graph of the function (35) intersects the axis of coordinates ($dr_*/dx = 0$) for the value of r_{*lim}.

The solutions of differential equations (24) and (35) can be obtained using the Runge-Kutta numeric integration method.

Separating the variables in differential equation (31) and integrating with the boundary conditions: $x = 0$ for $t_w = t_{wo}$ and $x = x_a$ for $t_w = t_{wa}$, abscissa x_a of cross-section in which the freezing begins ($r_* = R_* = R/R_f$), is determined by:

$$x_a = \frac{A}{C}\ln\frac{B - C(t_{wo} - t_e)}{B - C\left\{0.00559t_e\left(\frac{R_f}{G}\right)^{0.75}\frac{R_*^{0.75}}{\ln R_*^{0.75}} - 0.784\times10^{-3}\left[H_o + \left(J_p - 0.255\frac{n^2G^2}{R^{5.33}}\right)x_a\right] - t_e\right\}} \tag{36}$$

where:

$$A = 86400\,G\rho_w c_w$$

$$B = 22000\,\frac{\rho_w g n^2 G^3}{R^{5.33}} \tag{37}$$

$$C = \frac{86400 \times 2\pi}{0.0024\left(\frac{R}{G}\right)^{0.75} + \frac{1}{\lambda_g}\ln\frac{R}{R_f}}$$

Equation (36) can be solved by applying well-known numeric methods (secant method, Newton method, iteration method).

For $x = x_a$ and $x = L$ ($r_* = r_{*\lim}$), using the relation (33) are determined water temperatures t_{wa} and $t_{w\,lim}$, respectively.

By equating relations (1) and (3) written in differential form, after integration with the boundary conditions of the variables: $t_w = t_{wo}$ for $x = 0$ and $t_w = t_{wL}$ for $x = L$, the minimal discharge to prevent water freezing is obtained:

$$G_{min} = \frac{6.12\pi L (wR)^{0.7}}{\rho_w c_w \ln\frac{t_{wo} - t_e}{t_{wL} - t_e}} \tag{38}$$

in which: L is the pipe length; t_{wL} – water temperature in the final cross-section.

2.3.3 Variable ice layer depth in time along the pipe

However, generally, the ice layer formed on the pipe wall varies both in time and along the pipe: $r_* = r_*(x,\tau)$. In this case, the expressions (15) and (16) of the thermal balance equations take the forms:

$$72 \times 10^6 \pi (t_w + 0.784 \times 10^{-3} H)\left(\frac{G}{R_f}\right)^{0.75} \frac{1}{r_*^{0.75}} - \frac{86400\pi t_e}{\frac{1}{2\lambda_g}\ln r_*} - 2\pi\rho_g L_w R_f^2 r_* \frac{\partial r_*}{\partial \tau} = 0 \tag{39}$$

$$-86400 G\rho_w c_w \frac{\partial t_w}{\partial x} + 22000\frac{\rho_w g n^2 G^3}{R_f^{5.33}}\frac{1}{r_*^{5.33}} - 72 \times 10^6 \pi (t_w + 0.784 \times 10^{-3} H)\left(\frac{G}{R_f}\right)^{0.75} \frac{1}{r_*^{0.75}} = 0 \tag{40}$$

in which water pressure head H follows from (34).

By equation (39) the expression of the parameter t_w is determined, which then is derived with respect to x. Substituing in (40) the relations thus obtained and the values of known parameters, the second order differential equation results in:

$$A\frac{\partial^2 r_*}{\partial x \partial \tau} + B\frac{\partial r_*}{\partial x}\frac{\partial r_*}{\partial \tau} + C\frac{\partial r_*}{\partial x} + D\frac{\partial r_*}{\partial \tau} + E = 0 \tag{41}$$

where A, B, C, D and E are the functions of r_* and x defined as follows:

$$\begin{aligned}
A &= -3069.8 \times 10^9 G^{0.25} R_f^{2.75} r_*^{1.75} \\
B &= -5372.2 \times 10^9 G^{0.25} R_f^{2.75} r_*^{0.75} \\
C &= -2.02 \times 10^9 t_e G^{0.25} R_f^{0.75} \frac{0.75\ln r_* - 1}{r_*^{0.25}(\ln r_*)^2} \\
&\quad - 0.106 \times 10^{-6} \frac{G^3}{R_f^{5.33}} \frac{x}{r_*^{6.33}} \\
D &= -1.92 \times 10^9 R_f^2 r_* \\
E &= -1.264 \times 10^6 \frac{t_e}{\ln r_*} + 2.088 \times 10^5 \frac{G^3}{R_f^{5.33}} \frac{1}{r_*^{5.33}} \\
&\quad + 0.283 \times 10^9 J_p G
\end{aligned} \tag{42}$$

Ice layer variation in time along the pipe is described by the hyperbolic partial differential equation (41). The resolution of this equation consists in determining the function $r_* = r_*(x,\tau)$, which satisfies both the given equation and the initial conditions: $r_*(x,0) = R_* = R/R_f$ and $\frac{\partial r_*(x,0)}{\partial \tau} = 0$. The solution of the partial differential equation can be obtained using numerical finite-difference integration (grids method) by "crossways" procedure.

The values $r_{*i,j}$ for the function $r_*(x_i,\tau_j)$ are computed at the nodes (i,j) of a straightlines grid ($i = x/h$, $j = \tau/s$) from the plane $xO\tau$, where h and s are the length and the time step.

3. Numerical applications and results

Suppose a metallic outdoor pipe has the following characteristics: $G = 2.0$ m/s, $J_p = 0.015$, $R = 0.60$ m, $L = 4000$ m, $H_o = 10$ m, $t_e = -10$ °C, $w = 4.96$ m/s. The numerical simulation model developed above is used.

Fictitious radius is determined by the expression (7): $R_f = 0.60e^{0.615/(4.96 \cdot 0.6)^{0.7}} = 0.80$m, then, for ratio $R_* = R/R_f = 0.6/0.8 = 0.75$, using relation (27) the relative admissible radius results: $r_{*ad} = 0.705 \times 0.75 = 0.528$.

In the initial assumption of a constant external heat exchange on the circumference of the pipe a computer program is used to solve the differential equation (24), and we obtain $r_{*lim} = 0.529$ and $\tau_{lim} = 27$ days (for an admissible error of 0.00005). Also, the maximum admissible time of water stagnation in the pipe determined by the computer program is $\tau_{max} = 6.01$ days.

Based on the obtained numeric results, in figure 3 the variation diagram is represented for the ice layer during its forming, which has as asymptote the horizontal line $r_{*lim} = 0.529$. Figure 4 shows water temperature variation along the pipe for $\tau_{lim} = 27$ days. The following values are obtained for water temperature: 0.0190 °C in input cross-section ($x = 0$) and 0.0037 °C in the final cross-section ($x = L$). Therefore, after the time $\tau = \tau_{lim}$, for an input water temperature $t_{wo} = 0.0190$ °C, a constant ice layer along the pipe will be formed. This ice is characterized by the relative radius r_{*lim}.

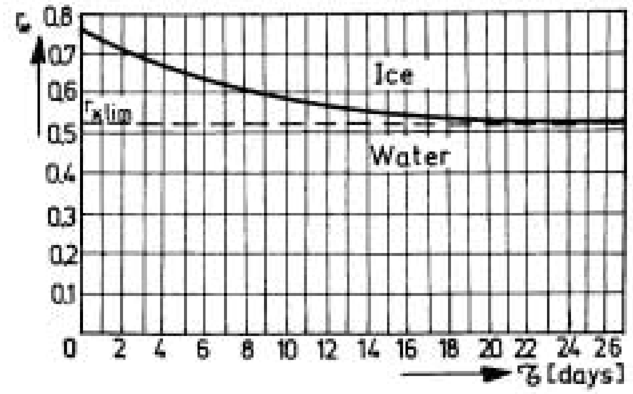

Fig. 3. Ice layer depth variation in time

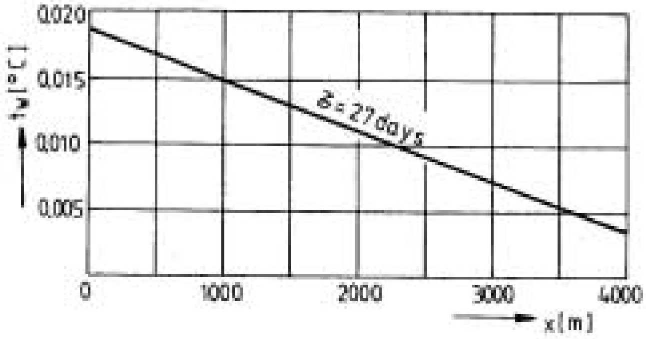

Fig. 4. Water temperature variation along the pipe with a constant ice layer depth (for $\tau = \tau_{lim}$)

Considering then an input water temperature $t_{wo} = 0.1$ °C and solving the differential equation (35) and as well as the equations (36) and (33) using the computer program, the following values are obtained: $r_{*lim} = 0.529$, $t_{wlim} = 0.0037$ °C for $x = L = 4000$ m and $x_a = 607.5$ m, $t_{wa} = 0.0646$ °C. Based on the computed results, in figures 5 and 6, the variation diagrams for ice layer, and water temperature along the pipe are presented.

Final numerical results obtained by solving the partial differential equation (41) in more than 2000 computational nodes, using a computer program, are represented in figure 7. Since $r_{*lim} = 0.529 > r_{*ad} = 0.528$, the pipe can function without thermal insulation. Water velocity in the pipe has the maximum value: $v_{max} = G/[\pi(R_f\, r_{*lim})^2] = 3.79$ m/s.

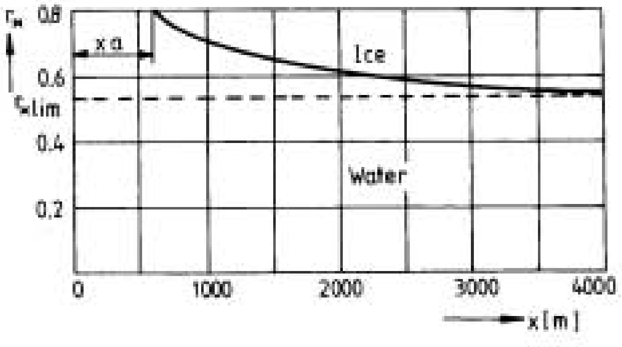

Fig. 5. Ice layer depth variation along the pipe ($t_{wo} = 0.1$ °C)

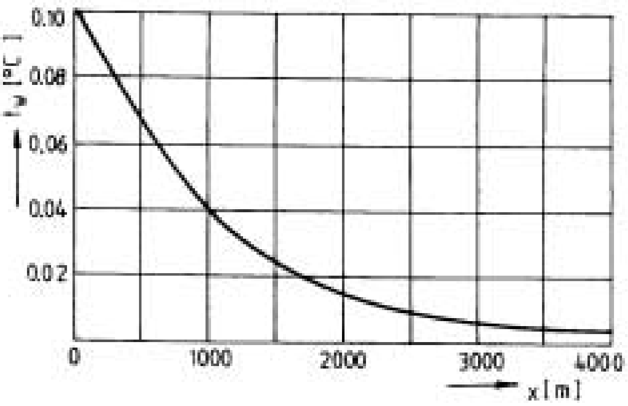

Fig. 6. Water temperature variation along the pipe with a variable ice layer depth (for $\tau = \tau_{lim}$)

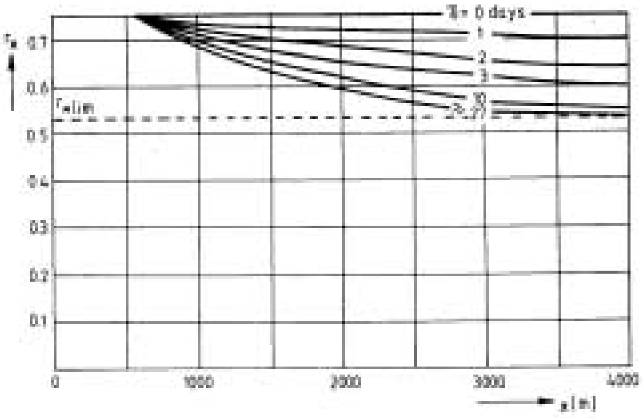

Fig. 7. Ice layer depth variation vs time along the pipe ($t_{wo} = 0.1$ °C)

Considering water temperature in the final cross-section $t_{wL} = 0$ °C, the minimal protection discharge $G_{min} = 3.96$ m^3/s is obtained by relation (38). For this discharge the relative limit radius and the water velocity have the following values: $r_{*lim} = 0.704$ and $v = 3.98$ m/s. Therefore, in this case, ice is formed on the pipe wall in a thin layer ($R\text{-}R_f\, r_{*lim} = 3.68$ cm) and as a result the flow velocity does not increase significantly (13.7 %), at given flow velocity in the pipe without ice ($v_o = 3.5$ m/s).

4. Conclusions

The numerical model for simulating variation in time along the pipe of ice layer in outdoor pressurized pipes has an increased generalization and accuracy level as compared to the relations determined in [7], for it considers also the disturbances of the external air flow.

Also, the model developed in this study offers the possibility to choose an optimal protection solution for the pipes against water freezing, based on the economic consideration, and to prevent this phenomenon in operation. Hence, it is recommendable to install along the pipes exposed to wind a control system formed of sufficient number of telethermometers with automatic signal and transmission, which indicates the moment when the water temperature reaches limit of 0 °C.

Notations

c_w – specific heat of water
dx – length of pipe element
G – discharge through pipe
G_{min} – minimum protection discharge
g – gravitational acceleration
H – water pressure head
H_o – available hydraulic head of pipe
J – hydraulic slope
J_p – pipe slope
L – pipe length
L_w – specific solidification heat of water
N – number of material layers
n – Manning roughness coefficient
R – inner radius of pipe
R_f – fictitious radius of pipe
r – radius of inside cross-section for discharge
r_* – relative radius
r_{*lim} – relative limiting radius corresponding to maximal ice layer depth
t_e – external air temperature
t_g – ice melting temperature
t_w – water temperature
t_{wo} – input water temperature
t_{wL} – water temperature in final cross-section
v – water velocity
w – wind velocity
x – abscissa of considered cross-section
x_a – abscissa of cross-section in which freezing begins
α_i, α_e – coefficients of internal and external heat convection
λ_j – thermal conductivity of material layer j of pipe
λ_g – thermal conductivity of ice
ρ_g – ice density
ρ_w – water density
τ – time elapsed
τ_{lim} – stabilization time of ice
τ_{max} – maximal admitted time for water stagnation

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