

# On the incipient aerated flow in chutes and spillways

## Sur le début d'entraînement d'air dans les écoulements de chutes et de déversoir

ANTONIO MOÑINO FERRANDO, *IAHR member, Group of Ports and Coastal Engineering, University of Granada, Granada, Spain.*

JAIME RIERA RICO, *IAHR member, Confederación Hidrográfica del Guadalquivir and School of Civil Engineering, Granada, Spain.*

### ABSTRACT

Natural self-aeration of water flows in open channels protects surfaces in contact with the flow from cavitation damage if enough air content is reached (Falvey [6] [7], Peterka [10], Russel & Sheenan [11]), although it could lead to an increase in both flow depth and velocity. Also, self-aerated flow enhance the process of atmospheric gases exchange within the fluid, improving water quality downstream of hydraulic structures (Chanson [5]). So it is of great interest to evaluate accurately the critical point where air entrainment begins, that is, the location of the inception point.

The note first shows a review on some methods to evaluate the inception point location. Second, a brief explanation on calculation development is made and an expression is obtained which enables to compute the point of incipient self-aeration as a function of unit discharge, bottom slope of the chute and uniform surface roughness in a more simple and accurate way. Finally, comparison with results provided by other methods is made and conclusions are obtained.

### RÉSUMÉ

L'auto aération de l'eau dans les canaux à ciel ouvert protège les surfaces en contact avec l'écoulement, des dommages dus à la cavitation, s'il y a suffisamment d'air (Falvey [6] [7], Peterka [10], Russel & Sheenan [11]), bien que cela puisse conduire à un accroissement de la hauteur d'eau et de la vitesse. L'auto aération renforce également le processus d'échange des gaz atmosphériques au sein du fluide, ce qui améliore la qualité de l'eau à l'aval des ouvrages hydrauliques (Chanson [5]). Il est donc fort intéressant d'évaluer avec précision le point critique du début d'entraînement d'air, c'est-à-dire la localisation du point d'amorçage.

Cette note présente d'abord quelques méthodes pour évaluer la position du point d'amorçage. On explique ensuite brièvement le développement du calcul puis on obtient une expression qui permet de calculer le point d'amorçage de l'auto aération en fonction du débit, de la pente du fond et de la rugosité uniforme de surface, d'une manière plus simple et précise. Finalement, on effectue une comparaison avec les résultats fournis par d'autres méthodes, et l'on en tire des conclusions.

## 1 Introduction

The inception point can be defined as the location  $\Delta x$ , measured along the channel invert (curvilinear coordinate), where air starts to enter into a body of water through the flow surface, beginning the process of natural self-aeration (see figure 1).

A first approach to the subject shows that, as a general rule, the natural aeration starts at the point where the developing boundary layer reaches the free surface of the flow, say, when the flow depth measured normal to the channel bottom equals the thickness of the boundary layer. However this is not true for flat slopes, in which the natural aeration might not begin at the same point where the outer limit of the boundary layer has reached the flow surface (Chanson [4]). So, two conditions should be present in order to ensure the air entrainment process: a turbulent regime is required and the energy of surface eddies must be higher than the energy content of the surface tension.

In fact, to determine the location of incipient self-aeration is to obtain the point where turbulence generated by the bottom shear forces reaches the flow surface, provided the channel section where the boundary layer starts to develop is correctly defined (it has been considered the spillway crest as the point where the boundary layer starts to grow, as shown in figure 1).

An interesting procedure to calculate the location of the inception point was proposed by Bormann [6]. It involves the solution of

equations for the loss coefficient, the Reynolds distance number and the drawdown curve. However, the method does not concern an explicit expression to evaluate the boundary layer thickness required to compute the loss coefficient. Instead of it, it is assumed that at the location of incipient air entrainment boundary layer thickness and flow depth -measured normal to the channel bottom- are equal. The calculating procedure is iterative, and it must be repeated until a distance is found so that the respective flow depth gives a simultaneous solution for the above mentioned equations.

The method proposed in [8] is more direct than Bormann's and

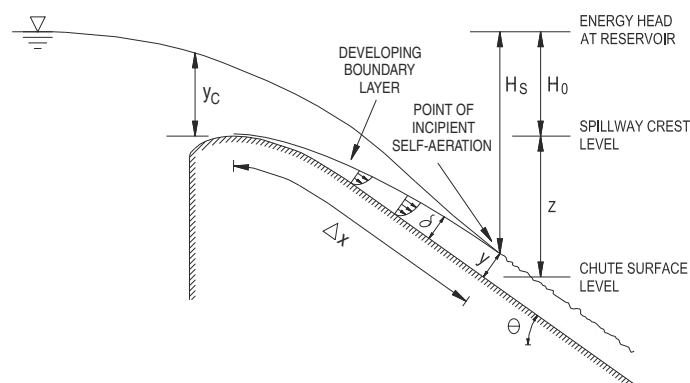


Fig. 1 Location of the inception point and main parameters involved in the study.

Revision received September 6, 2000. Open for discussion till August 31, 2002.

it has been used as the procedure to develop the analysis here presented. Essentially, the method consists of the simultaneous solving of equations for the drawdown curve and the growing boundary layer, regarding the main feature of the method is the use of Wood's expression to compute the boundary layer thickness  $\delta$  (Wood, Ackers & Loveless [14]):

$$\frac{\delta}{\Delta x} = 0.0212 \left( \frac{\Delta x}{H_s} \right)^{0.11} \left( \frac{\Delta x}{K_s} \right)^{-0.10} \quad (1)$$

which accounts for the energy head  $H_s$  on the flow surface at the given location (see figure 1):

$$H_s = H_0 + z - y \cos \theta \quad (2)$$

and for the value of the equivalent uniform roughness  $K_s$ , both of them factors affecting the boundary layer growth.

## 2 Calculations

A multiple regression analysis has been applied to the results provided by the late method described in the previous section. The analysis has been conducted for a prismatic channel with rectangular cross section, applying a set of unit discharges  $q$  varying from  $0.5 \text{ m}^3/\text{s}\cdot\text{m}$  to  $20 \text{ m}^3/\text{s}\cdot\text{m}$ , slope angles  $\theta$  between  $5^\circ$  and  $70^\circ$ , and equivalent sand grain roughness  $K_s$  from  $0.001 \text{ m}$  to  $0.003 \text{ m}$ , so that it is covered a wide range of supercritical flow conditions and surface characteristics which are present in hydraulic structures. The results here presented should be applied to channels with steep slopes and supercritical regime flow, in which velocity conditions could lead to cavitation damage (i.e. when flow velocity is greater than  $15 \text{ m/s}$ ) if enough air content is not reached. In this sense, the location of the inception point plays an essential role in the performance of the discharge along the chute. No data are obtained for flat slopes with subcritical flows since in most practical cases velocity regime does not approach a dangerous limit for cavitation damage.

First, for each given value of  $K_s$  and  $\theta$  it has been obtained an expression which relates the location  $\Delta x$  of the inception point with the unit discharge. Second, for the same fixed values of  $K_s$  the dependence with the slope angle  $\theta$  has been studied, so that a set of expressions relating the location of the inception point, the unit discharge and the slope angle have been carried out (one expression for each single value of  $K_s$ ). Third, the dependence with roughness has been analyzed, and a final expression regarding the relationship between  $\Delta x$ ,  $q$ ,  $\theta$  and  $K_s$  has been developed. This expression enables to evaluate the location of the inception point for given values of  $q$ ,  $\theta$  and  $K_s$  by means of a direct computing. Its functional form is as follows:

$$\Delta x = \left( \frac{q}{0.05642 K_s^{0.056} (\sin \theta)^{0.34}} \right)^F \quad (3)$$

where exponent  $F$  is:

$$F = (1.46443 K_s^{0.0054} (\sin \theta)^{0.0027})^{-1} \quad (4)$$

The agreement between the data provided by the previous expressions and the numerical solutions for the simultaneous solving of the drawdown curve and the boundary layer thickness is excellent. Thus, the results from (3) show a maximum difference of  $0.22\%$  greater than the numerical results for  $\theta \approx 70^\circ$  and  $q \approx 20 \text{ m}^3/\text{s}\cdot\text{m}$ , while for  $5^\circ \leq \theta < 70^\circ$  and  $q < 20 \text{ m}^3/\text{s}\cdot\text{m}$  the absolute difference is not greater than  $0.1\%$ . It must be kept in mind that expression (3) is only valid for rectangular chutes with supercritical flows in which the start point of the boundary layer growth is well known.

## 3 Discussion

Other expressions for the location of the inception point have been proposed by several authors. In this sense, Wood, Ackers & Loveless [13] obtained, as a previous stage to their final result for the boundary layer thickness, an equation for the incipient self-aeration based upon Keller & Rastogi theoretical results. Obviously, this equation did not take into account the boundary layer growth as proposed by them later.

Also, in an interesting work of Hager & Blaser [9] it is found an expression for the beginning of air entrainment, developed from an analytical solution for the drawdown curve. Their expression involves the use of a modified Bauer's equation for the boundary layer thickness.

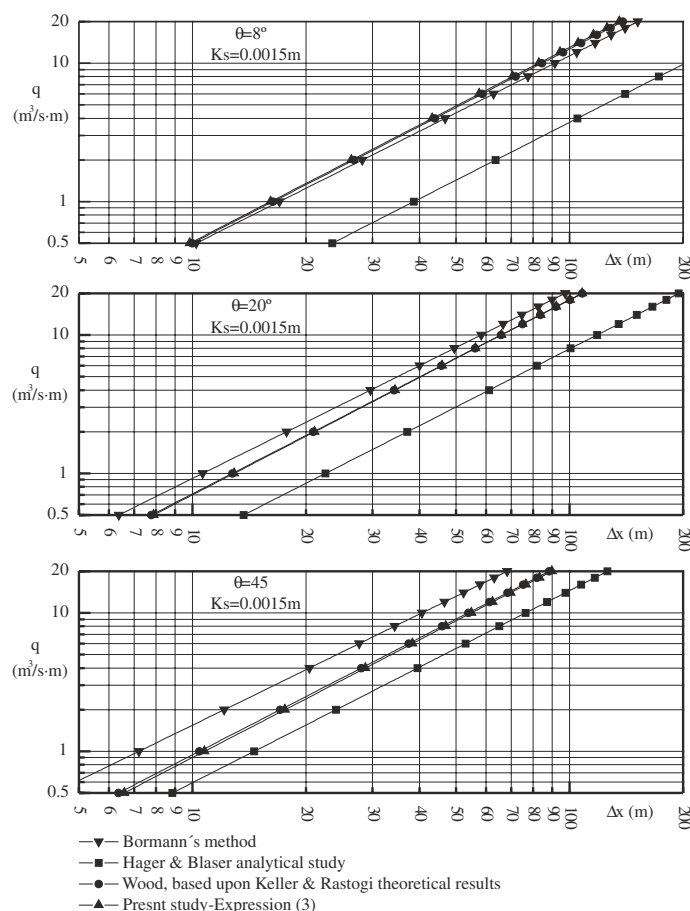


Fig. 2 Comparison of data provided by present study (expression (3)) and other authors.

Figure 2 shows the results of applying Bormann's method, Wood's formula based upon Keller & Rastogi results, the expression suggested by Hager & Blaser and expression (3), for various slope angles and a value of  $K_S$  equal to 0.0015m.

It can be observed in the graphics that results from Hager & Blaser give a distance to the beginning of air entrainment greater than the predictions of expression (3), although differences are reduced as the slope angle increases. This is due to the fact that the boundary layer thickness as computed by Bauer's modified formula is smaller than the result obtained by Wood's expression (1) for the same flow and geometrical conditions, being the difference in some cases of about 30%. So, for a given discharge, a smaller value for the boundary layer thickness at a given position along the chute invert results in a greater distance required for the outer edge of the boundary layer to reach the flow surface, thus moving the inception point further downstream.

On the other hand, Bormann's method provides locations of the inception point slightly greater than data provided by (3) in the case of small angles, but as the slope increases, say for  $\theta \geq 10^\circ$ , this trend is reversed, and results from (3) remain far downstream than those by Bormann's method.

Finally, the data provided by equation (3) are in good agreement with the calculations by Wood's formula based upon Keller & Rastogi results, for all discharges and slope angles.

#### 4 Conclusions

Expression (3) provides the location of incipient air entrainment into the flow in a rectangular chute, for given values of discharge, slope angle and uniform roughness. Its reliability is based on the fact that Wood's expression for the boundary layer thickness has been used in its development. However, for flat slopes the predictions might differ from the real location of the inception point (Chanson [4], Falvey [6]). Both experimental and prototype measurements would be required so that a corrective factor could be added to the expression here proposed, in order to take into account that effect.

#### 5 Acknowledgements

Authors wish to thank Dr. Willi H. Hager (Laboratory of Hydraulics, Hydrology and Glaciology, Zurich, Switzerland) and

Dr. Hubert Chanson (The University of Queensland, Australia) for their kind long-distance help and interesting advices.

#### 6 References

1. BAUER, W.J., 1954. Turbulent boundary layer on steep slopes. *Transactions*, ASCE, Vol. 119, Paper n° 2719, pp. 1212-1233.
2. CHANSON, H., 1989. Study of air entrainment and aeration devices. *Journal of Hydraulic Research*, Vol.27, n° 3.
3. CHANSON, H., 1989. Flow downstream of an aerator-aerator spacing. *Journal of Hydraulic Research*, Vol.27, n° 4.
4. CHANSON, H., 1997. Air bubble entrainment in open channels. Flow structure and bubble size distributions. *International Journal of Multiphase Flow*, Vol. 23, n° 1, pp. 193-203.
5. CHANSON, H., 1999. Aeration performance of open chutes. *Journal of Hydraulic Engineering, Discussion*, pp. 666-667.
6. FALVEY, H.T., 1980. Air water flow in hydraulic structures. *Water Resources Technical Publication*, Engineering monograph n° 41, U.S. Dept. of Interior, U.S. Printing Office, Denver, Colorado.
7. FALVEY, H.T., 1990. Cavitation in chutes and spillways. *Water Resources Technical Publication*, Engineering monograph n° 42, U.S. Dept. of Interior, U.S. Printing Office, Denver, Colorado.
8. HAGER, W.H., 1992. Spillways, shock waves and air entrainment. *Bulletin 81*, I.C.O.L.D.
9. HAGER, W.H. & BLASER, F., 1998. Drawdown curve and incipient aeration for chute flow. *Canadian Journal of Civil Engineering*, 25, pp. 467-473.
10. PETERKA, A.J., 1955. The effect of entrained air on cavitation pitting. *V.I.A.H.R. Congress*, Minnesota, pp. 507-518.
11. RUSSELL, S.O. & SHEENAN, G.J., 1974. Effect of entrained air on cavitation damage. *Canadian Journal of Civil Engineering*, Vol. 1.
12. STRAUB, L.G. & ANDERSON, A.G., 1958. Experiments on self aerated flow in open channel. *Journal of Hydraulic Division, ASCE*, Vol. 84, HY7.
13. WOOD, I.R., ACKERS, P. & LOVELESS, J., 1983. General method for critical point on spillways. *Journal of Hydraulic Engineering*, Vol. 109, n° 2, pp. 308-312.