Leak detection in pipes by frequency response method using a step excitation

Détection des fuites de tuyauteries par une méthode de réponse en fréquence utilisant un échelon d'excitation

WITNESS MPESHA, *Graduate Student*

M. HANIF CHAUDHRY, *Mr. & Mrs. Irwin B. Kahn Professor*

SARAH L. GASSMAN, *Assistant Professor*

Department of Civil and Environmental Engineering, University of South Carolina Columbia, SC 29208

Abstract
This paper presents a new procedure utilizing transient state pressures to detect leakage in piping systems. Transient flow, produced by opening or closing a valve, is analyzed in the time domain by the method of characteristics and the results are transformed into the frequency domain by the fast Fourier transform. This method is used to develop a frequency response diagram at the valve end. The frequency response diagram of a system with leaks has additional resonant pressure amplitude peaks (herein called the secondary pressure amplitude peaks) that are lower than the resonant pressure amplitude peaks for the system if there were no leaks (herein called primary amplitude peaks). The location of a leak is determined from frequencies of the primary and secondary pressure amplitude peaks and the leak discharge is determined from the maximum and minimum discharge amplitudes. This method is applicable for practical values of the friction factor over the range 0.01 to 0.025 and can be used to detect leaks in real-life pipe systems conveying different types of fluids, such as water and petroleum. It can be used directly by comparing the frequency response diagram of a modeled system without leaks to the frequency response diagram developed by gradually opening or closing a valve at the downstream end of a pipe and taking measurements of pressure head and discharge at only one location.

Résumé
Cet article présente un nouveau procédé de détection de fuites dans les tuyauteries, qui utilise les pressions instationnaires. L'écoulement transitoire, provoqué par l'ouverture ou la fermeture d'une vanne, est analysé en fonction du temps par la méthode des caractéristiques et les résultats sont transposés dans le domaine des fréquences par une transformée de Fourier rapide. Cette méthode est utilisée pour établir un diagramme de réponse en fréquence à la vanne d'extrémité. Le diagramme de réponse en fréquence d'un système avec fuites présente des pics de résonance additionnels en amplitude de pression (nommés ici pics secondaires d'amplitude de pression) qui sont plus faibles que les pics de résonance du système sans fuites (nommés pics primaires d'amplitude). La localisation d'une fuite est déterminée à partir de la fréquence des pics primaires et secondaires d'amplitude de pression, et le débit de fuite est déterminé à partir des amplitudes maximum et minimum de débit. Cette méthode s'applique pour des valeurs pratiques du coefficient de frottement entre 0.01 et 0.025, et peut être utilisée pour détecter les fuites des systèmes de conduites en fonctionnement transportant différents types de fluides comme l'eau et le pétrole. On peut l'utiliser directement en comparant le diagramme de réponse en fréquence d'un système modèle sans fuite au diagramme développé par l'ouverture ou la fermeture graduelle d'une valve à l'extrémité aval d'un tuyau en mesurant la pression et le débit en un seul endroit.

Introduction

Leaks in piping systems pose a major operational problem around the world. Leaks may occur due to poor quality and defective pipe materials, pipe breaks resulting from poor workmanship, operational errors such as excessive pressure, closing or opening valves rapidly, corrosion, leaking fittings and accidental or deliberate damage to fixtures (AWWA 1990). This results in economic loss, safety and environmental issues.

Detecting, locating and repairing these leaks become a painstaking task. In many cases, leakage receives little or no attention from consumers or the media and there is often no leakage management program until an emergency occurs or when there is shortage. Different methods for leak detection have been developed. Leak detection methods that are based on transient analysis such as Liou (1990), Nicholas (1990), Pudar and Liggett (1992), Liou and Tian (1994), Liggett and Li-Chung (1994), Silva et al. (1996) and Liou (1998) usually are based on the acquisition and analysis of extensive real-time data. Often, these data are either not available or costly to obtain.

In previous work (Mpesha, 1999), the authors developed a method that uses the frequency response obtained by analyzing steady-oscillatory flow in a pipe system. An oscillating valve located at the end of the pipeline is used to produce steady oscillatory flow in the system. The steady-oscillatory flow is analyzed in the frequency domain by the transfer matrix method and a frequency response diagram is developed from which leaks are detected based on the pressure and discharge amplitude peaks.

In this paper, a leak detection method is presented that uses frequency response obtained for a non-periodic excitation. The method of characteristics is used to analyze the transients created in the system by opening or closing a valve. The results of the analysis in the time domain are transformed to the frequency domain by using the fast Fourier transform. The method of characteristics is briefly outlined first, followed by a brief discussion of the theory of fast Fourier transform and its application to leak detection. Typical pipeline systems are examined and the results are presented and discussed.

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Analysis of pipe transients in the time domain by the method of characteristics

The method of characteristics is widely used in solving the hyperbolic, partial differential equations describing transient-state flow in pipes. When the continuity and momentum equations describing unsteady flow in pipes are linearly combined and the resulting equation is integrated, the following equations are obtained (Chaudhry 1987):

$$Q_P - Q_A + \frac{gA}{a}(H_P - H_A) + R\int_A^P Q|Q|dt = 0 \qquad (1)$$

$$Q_P - Q_B + \frac{gA}{a}(H_P - H_B) + R\int_B^P Q|Q|dt = 0 \qquad (2)$$

where Q_A, Q_B and Q_P are instantaneous discharges at A, B and P respectively, H_A, H_B and H_C are instantaneous pressure heads at A, B and P respectively, g = gravitational acceleration, A = cross-sectional area of pipe, a = wave velocity, and R = f /2DA. Eq. 1 is valid along the positive characteristic line AP (Fig. 1), and Eq. 2 is valid along the negative characteristic line BP.

Approximation equation for the friction-loss term

Since the variation of discharge Q with respect to time t is not known explicitly, the integral in the last term of Eqs. 1 and 2 (the friction-loss term) has to be approximated. There are several ways to approximate the friction-loss term. For the positive characteristic line AP, $R\int_A^P Q|Q|dt$ is approximated by the following equation

$$R[Q_A + \varepsilon(Q_P - Q_A)]\,|Q_A|\Delta t \qquad (3)$$

Similar equations can be written for the negative characteristics line BP (Fig.1) by substituting subscript A with B. For this work, ε=0.85 was used because it gives the most accurate results

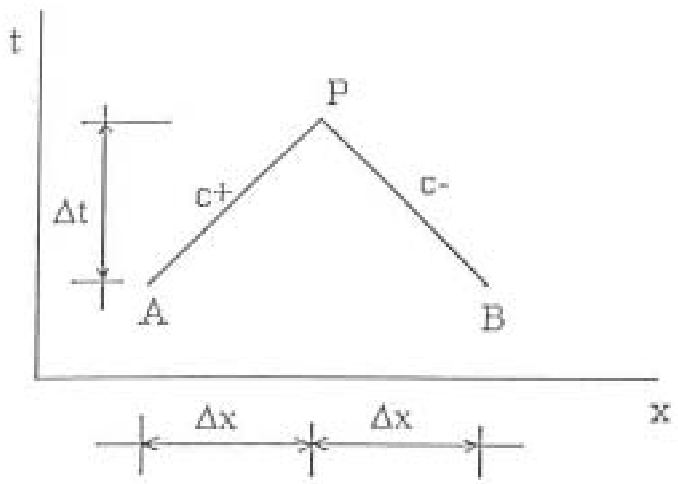

Fig. 1 Characteristic Grid

(Karney & McInnis, 1992).
Chaudhry (1987) developed various special boundary conditions. Some of these boundary conditions are used in this work, which includes, constant head reservoir at upstream end, series junction, branching junction, valve at downstream end and dead end. The boundary condition for a leak is developed herein.

The Fast Fourier transform

In the system considered, a valve at the downstream end is opened or closed gradually thus producing waveforms h(t) for the pressure heads and q(t) for the discharges. Both h(t) and q(t) are non-periodic. The results obtained by analyzing the transients by the method of characteristics represent the desired waveforms h(t) and q(t). The Fourier transform is used as a problem-solving technique to transform waveforms h(t) and q(t) into the frequency domain as shown in the following equation

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi\omega t}dt \qquad (4)$$

where h(t) is the waveform function of variable time t representing the pressure head and the discharge. H(ω) is the function of variable frequency ω and $j = \sqrt{(-1)}$. H(ω) is the Fourier transform of h(t); thus h(t) and H(ω) are a continuous transform pair.
The Fourier transform pair has to be suitably modified to develop a discrete Fourier transform that is acceptable for computations (Brigham, 1988). The procedure for this modification is outlined as follows:

1. Sample the waveform h(t) by multiplying it with a time-domain sampling function.
2. Truncate the sampled function by multiplying it with a rectangular function.
3. Sample the Fourier transform of the result in step 2 to produce a periodic function.
4. Develop the Fourier transform of the periodic function obtained in step 3. This step gives the desired discrete Fourier transform, which is an approximation to the continuous Fourier transform H(ω).
5. Compute the discrete Fourier transform by the fast Fourier transform (FFT).

The FFT is an algorithm used to compute the discrete Fourier transform rapidly. It involves factorization and reversing the binary numbers, which reduces the number of multiplications.

Application to leak detection

Boundary condition for a leak

To analyze transients in pipes by the method of characteristics for the purpose of detecting leaks, it is necessary to develop the boundary condition for a leak. The boundary condition for a leak as shown in Fig. 2, can be developed as follows:

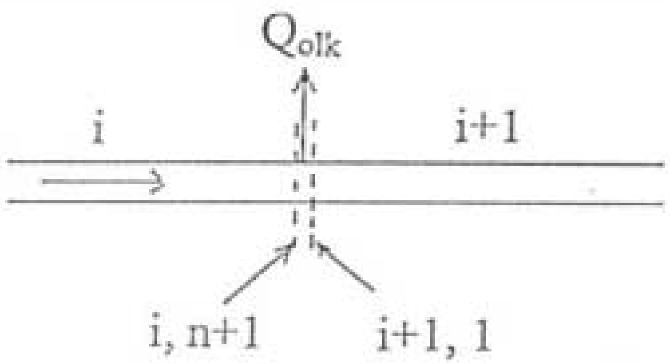

Fig. 2 Leak Boundary

Continuity Equation

$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}} + q_{olk} \tag{5}$$

Total head equation

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} \tag{6}$$

Characteristic equations
Positive characteristic

$$Q_{P_{i,n+1}} = C_{p_i} - C_{a_i} H_{P_{i,n+1}} \tag{7}$$

Negative characteristic

$$Q_{P_{i+1,1}} = C_{n_{i+1}} + C_{a_{i+1}} H_{P_{i+1,1}} \tag{8}$$

where

$$C_p = Q_A + \frac{gA}{a} H_A - R\Delta t\left[Q_A + \varepsilon\left(Q_P - Q_A\right)\right]\left|Q_A\right| \tag{9}$$

$$C_n = Q_B + \frac{gA}{a} H_B - R\Delta t\left[Q_B + \varepsilon\left(Q_P - Q_B\right)\right]\left|Q_B\right| \tag{10}$$

$$C_a = \frac{gA}{a} \tag{11}$$

Solving Eqs. 5-8 simultaneously,

$$H_{P_{i,n+1}} = \frac{C_{p_i} - C_{n_{i+1}} - q_{olk}}{C_{a_{i+1}} + C_{a_i}} \tag{12}$$

To complete the solution, Eq. 12 is then used to determine the remaining quantities $H_{P_{i+1,1}}, Q_{P_{i,n+1}}$ and $Q_{P_{i+1,1}}$. The results obtained by analyzing the transients by the method of characteristics represent the desired waveforms h(t) for the heads and q(t) for the discharges. The system setup is such that a valve at the pipe end is closed gradually thereby producing waveforms h(t) and q(t) which are non-periodic. These non-periodic waveforms and their Fourier transforms have to be modified according to the process explained in the previous section, i.e., sampling the waveforms h(t) and q(t), truncating the sampled function, sampling the Fourier transforms, developing the discrete Fourier transform and finally, computing the discrete Fourier transform by the FFT.

The computation of the discrete Fourier transform by the FFT produces the desired H(ω) and Q(ω), where H(ω) is the head with respect to frequency and Q(ω) is the discharge with respect to frequency, both at the valve end. The frequency diagram is then developed as h_r, q_r versus ω_r, where $h_r = H(\omega)/H_o$ is the pressure head ratio (non-dimensional pressure amplitude), $q_r = Q(\omega)/Q_o$ is the discharge ratio (non-dimensional discharge amplitude) and $\omega_r = \omega / \omega_{th}$ is the frequency ratio. The theoretical frequency ω_{th} is $2\pi / T_{th}$ where the theoretical period of the pipeline T_{th} is given by the sum of the period of the pipe components with lengths L_1, L_2,…L_n, i.e.,

$$T_{th} = \frac{4L_1}{a_1} + \frac{4L_2}{a_2} + \frac{4L_3}{a_3} + \ldots + \frac{4L_n}{a_n} = \sum_{i=1}^{n} \frac{4L_i}{a_i} \tag{13}$$

In previous work (Mpesha, 1999), the authors established the following equation for obtaining the location of the leak from the frequency response diagram

$$L_i = \frac{a_i}{\frac{3}{4}\Delta\omega_{r_i}} \tag{14}$$

where L_i is the length of the pipe, a_i is the wave velocity in the pipe corresponding to L_i, and $\Delta\omega_{ri}$ is the change in the frequency ratio between resonant peaks of non-dimensional pressure amplitudes on the frequency response diagram corresponding to L_i. To obtain the total length L of the pipe for the ‘‘no leak’’ case, the following equation was also established

$$L = \sum_{i=1}^{n} \frac{a_i}{\frac{1}{8}\Delta\omega_{r_i}} \tag{15}$$

where n is the number of components in the pipe system.

The leak discharge was obtained by using the following equation

$$q_{olk} = \frac{1}{2} Q_o \left(q_{r_{max}} - q_{r_{min}}\right) \tag{16}$$

Eqs. 14-16 are observed to be applicable in this method.

Examples of application to leak detection

To illustrate the applicability of the method, the effect of the leak on the frequency response at the valve end was analyzed for different piping systems and is discussed in the following paragraphs.

Single pipe system

1. ONE LEAK

Figure 3 shows a single pipe system with one leak introduced at a given location along the pipeline. For the case evaluated here, the leak discharge, $q_{olk} = 0.0069$ m^3/s, diameter of leak, $d_{lk} = 0.02$ m, pipe diameter, D = 0.3046 m, H_o = 50 m, $Q_o = 0.1$ m^3/s, C_d = 0.7, a = 1200 m/s, L_1 = 400 m, and L_2= 1200 m. The leak is equal to 6.9% of the mean discharge through the pipe.

Fig. 4 shows the frequency response diagram at the valve end for the system shown in Fig. 3. By comparing the frequency responses of the system with a leak to the system without a leak, the occurrence of a leak is indicated by the presence of a secondary pressure amplitude peak ($(h_{r\,max})_{lk}$) in between the primary pressure amplitude peaks ($h_{r\,max}$) of the system without a leak. Using these resonant peaks, the change in frequency ratio $\Delta\omega_{r1} = 4$ and $\Delta\omega_{r2} = 2$. Substituting these values into Eq. 14 gives the length of pipe to the leak as L_1 = 400 m. Substituting $\Delta\omega_{r1} = 4$ in Eq. 15 gives the total length of the pipe as 1600 m. This corresponds to the actual leak location and total length in Fig.3. Also, it is noted that $q_{r_1\,max}$ and $q_{r_1\,min}$. Substituting these values into Eq. 16, the leak discharge is obtained as 0.0069 m^3/s, which corresponds to the actual leak discharge.

2. MULTIPLE LEAKS

Leaks were introduced at two locations along the pipeline as shown in Fig. 5. The diameter of leak 1,d_{lk_1}= 0.011 m, diameter of leak 2, d_{lk_2}= 0.017 m, pipe diameter, D = 0.3046 m, H_o = 50 m, Q_o = 0.1 m^3/s, C_d = 0.7, a = 1200 m / s, L_1 = 355.5 m, L_2 = 640 m, and L_3 = 133.9 m.

Similarly, Eq. 14 is applicable for multiple leaks. Fig. 6 shows the frequency response diagram at the valve end for the system shown in Fig. 5. By comparing the frequency responses of the system with two leaks to the system without a leak, the occurrence of two leaks is indicated by the presence of two lower pressure amplitude peaks, $(h_{r\,max})_{lk1}$ and $(h_{r\,max})_{lk2}$, in between two higher primary pressure amplitude peaks ($h_{r\,max}$). From Fig. 6, $\Delta\omega_{r1} = 4.5$, $\Delta\omega_{r2} = 2.5$, and $\Delta\omega_{r3} = 1.5$. Substituting these values into Eq. 14 gives the length of the pipe to the leaks as L_1= 355.5 m, and L_2= 640 m. These values correspond to the actual leak locations in Fig. 5. The leak discharge can also be calculated for the multiple leak case using Eq. 16. Note that $q_{r_1\,max}$ and $q_{r_1\,min}$

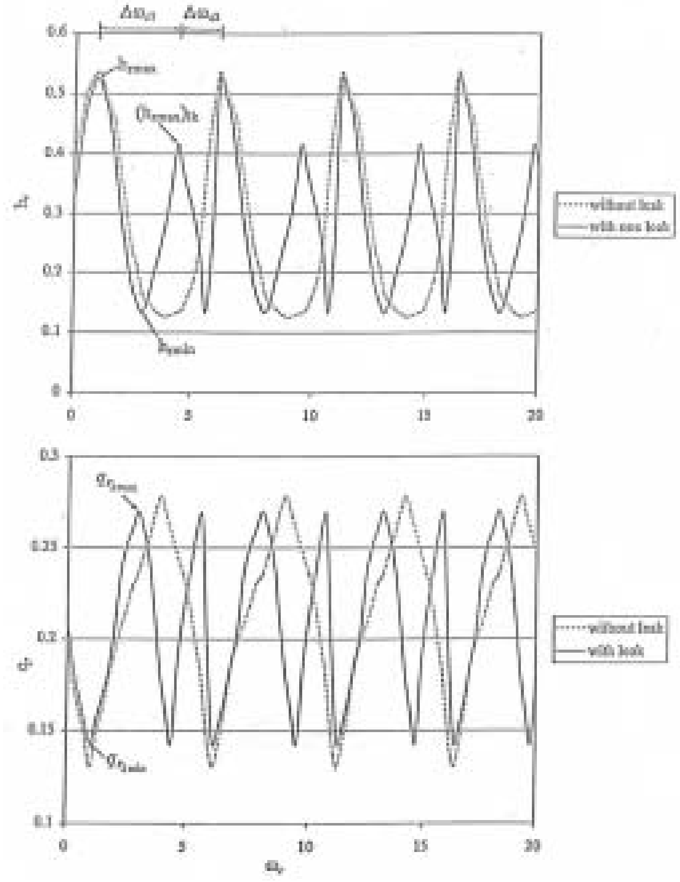

Fig. 4 Frequency Response for the System with One Leak Shown in Fig. 3.

correspond to $\Delta\omega_{r_1}$ and $q_{r_2\,max}$ and $q_{r_2\,min}$correspond to $\Delta\omega_{r_2}$. From Fig. 6, $q_{r_1\,max}$ and $q_{r_1\,min}$. Substituting these values into Eq. 16 gives the leak discharge q_{olk_1} = 0.0021m^3/s. Also, $q_{r_2\,min}$= 0.278 and $q_{r_2\,min}$= 0.178. This gives the leak discharge q_{olk_2} = 0.005m^3/s. These computed values are the same as the actual leaks of Fig. 5.

Parallel pipe system

In Fig. 7, one leak is introduced on a parallel pipe system. The leak discharge, q_{olk} = 0.005 m^3/s, which is equal to 5% of the mean discharge through the pipe**,** the diameter of leak, d_{lk} = 0.017

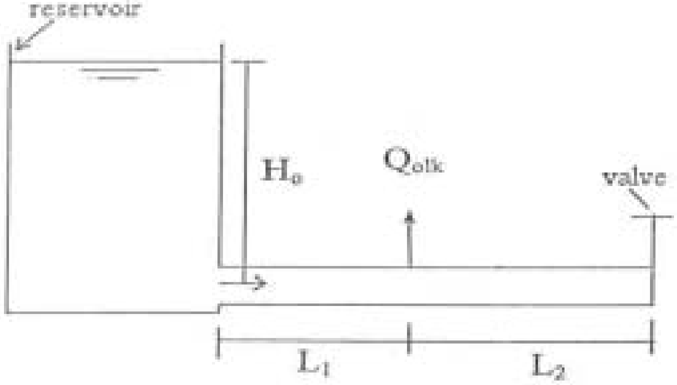

Fig. 3 Single Pipe with One Leak

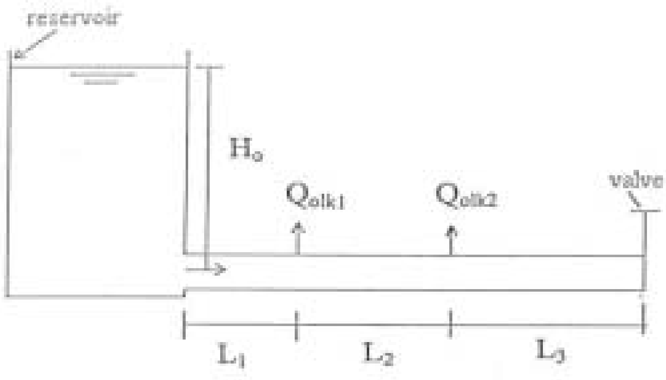

Fig. 5 Single Pipe with Multiple Leaks.

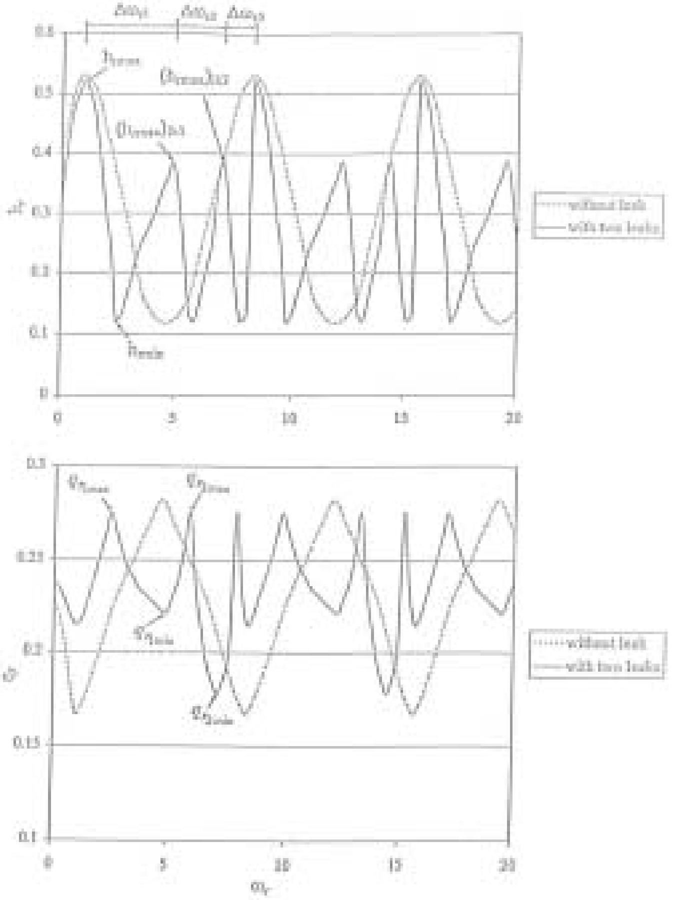

Fig. 6 Frequency Response for the System with Two Leaks Shown in Fig. 5.

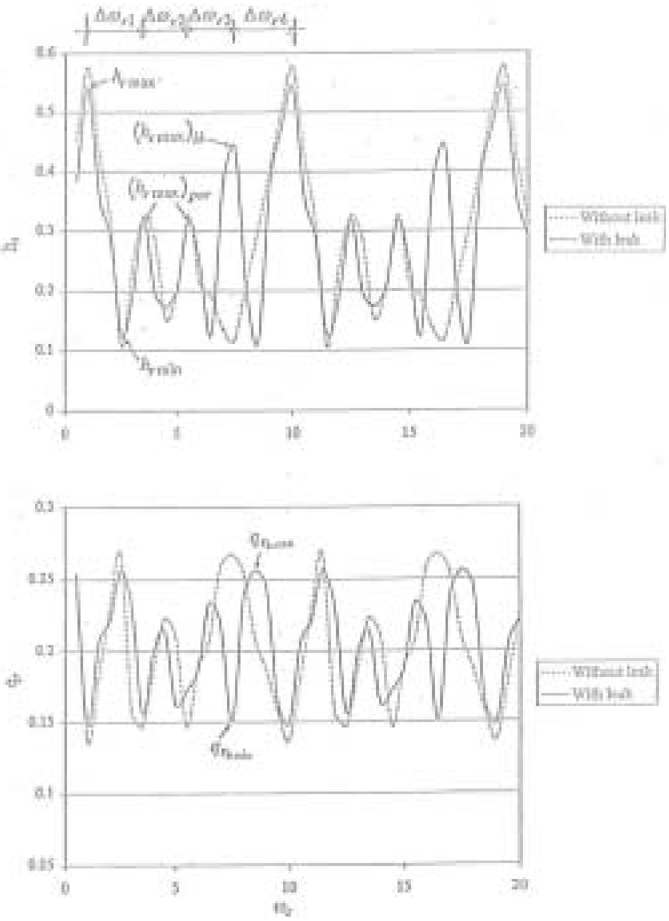

Fig. 8 Frequency Response for the Parallel Pipe System Shown in Fig. 7

m, pipe 1 diameter, D_1 = 0.3046 m, pipe 2 diameter, D_2 = 0.256 m, pipe 3 diameter, D_3 = 0.256 m, pipe 4 diameter, D_4 = 0.3046 m, H_o = 50 m, Q_o = 0.1 m^3/s, C_d = 0.7, a_1 = 937 m/s, a_2 = a_3 = 1200 m/s, a_4 = 1275 m/s, L_1 = 499.7 m, L_2 = L_3 = 800 m, and L_{4a} = 850 m, L_{4b} = 7915.3 m.

It is observed that Eq. 14 is valid for determining the location of the leak and the branching points for the parallel pipes. Fig. 8 shows the frequency response diagram at the valve end for the parallel pipe system of Fig. 7. It is observed that in between two primary pressure amplitude peaks ($h_{r\,max}$)corresponding to the system without a leak, there is one lower pressure amplitude peak and two lowest amplitude peaks. By comparing the frequency responses of the case with and without a leak, it is clear that the lower pressure amplitude peak $(h_{r\,max})_{lk}$ indicates the presence of a leak and the two lowest amplitude peaks $(h_{r\,max})_{par}$ corresponds to the branching points for the parallel pipes. From Fig. 8, $\Delta\omega_{r1}$ = 2.5, $\Delta\omega_{r2}$ = 2, $\Delta\omega_{r3}$ = 2 and $\Delta\omega_{r4}$ = 2.5. The lengths of pipe to the leak and the branching points are determined from Eq. 14 to be L_1 = 499.7 m, L_2 = 800 m, L_3 = 800 m and L_{4a}= 850 m. These values correspond to the actual leak and branching point locations in Fig. 7. The leak discharge (Eq. 16) is also observed to be applicable for this system. Note that $q_{r_4\,max}$ and $q_{r_4\,min}$ correspond to $\Delta\omega_{r_4}$. From Fig. 8, $q_{r_4\,max}$= 0.2501 and $q_{r_4\,min}$ = 0.1501. Substituting these values in Eq. 16, the leak discharge is obtained as q_{olk} = 0.005 m^3/s and corresponds to the actual value of the leak in Fig. 7.

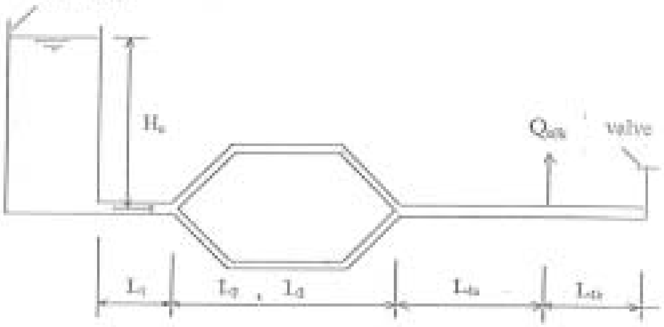

Fig. 7 Parallel Pipe System with One Leak.

Branched pipe system

Fig. 9 shows a system consisting of a branched pipe with a dead end and a series junction on the main pipe. One leak is introduced on the main pipe. The leak discharge, q_{olk} = 0.0069 m^3/s, which is equal to 6.9% of the mean discharge through the pipe**,** the diameter of leak, d_{lk} = 0.02 m, pipe 1 diameter, D_1 = 0.3046 m, branched pipe diameter, D_2 = 0.256 m, pipe 3 diameter, D_3 = 0.256 m, H_o = 50 m, Q_o = 0.1 m^3/s, C_d = 0.7, a_1 = a_2 = a_3 = a_4 = 950 m/s, a_2 = 1150 m/s, L_1 = 633.3 m, L_2 = 506.7 m, L_3 = 361.9 m, L_4 = 11164.8 m, and L_{br} = 1022.2 m.

It is observed that Eq. 14 is valid for determining the location of

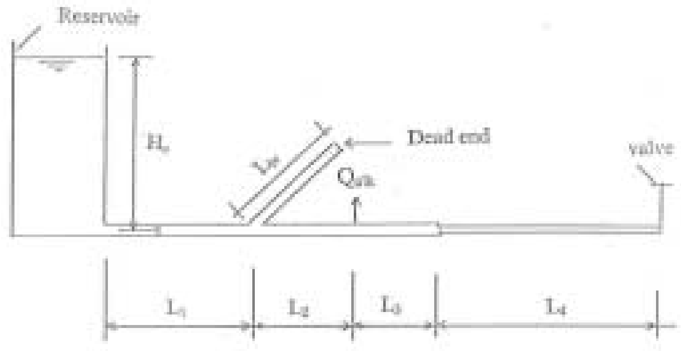

Fig. 9 Branched Pipe System with One Leak.

the leak and the branched pipe. Fig. 10 shows the frequency response diagram at the valve end for the branched pipe system of Fig. 9. It is noted that there are three lower pressure amplitude peaks in between two primary pressure amplitude peaks ($h_{r\,max}$) corresponding to the system without a leak. By comparing the frequency responses of the case with and without a leak, it is clear that the highest of these three pressure amplitude peaks $(h_{r\,max})_{lk}$ indicates the presence of a leak, the lower amplitude peak $(h_{r\,max})_{sj}$ corresponds to the series junction and the lowest amplitude peak $(h_{r\,max})_{br}$ corresponds to the branched pipe junction. From Fig. 10, $\Delta\omega_{r1} = 2$, $\Delta\omega_{r2} = 2.5$, $\Delta\omega_{r3} = 3.5$ and $\Delta\omega_{r4} = 1$. The lengths of pipe to the leak, branched pipe and series junctions are determined from Eq. 14 to be $L_1 = 633.3$ m, $L_2 = 506.7$ m, and $L_3 = 361.9$ m, and $L_4 = 11164.8$ m. Substituting $\Delta\omega_{r1} = 2$, $(\Delta\omega_{r2} + \Delta\omega_{r3})$ 2.5 + 3.5 in Eq. 15 gives the total length of the pipe as 12666.7 m. These values correspond to the actual leak, branched pipe, series junctions locations and total length in Fig. 9. The leak discharge (Eq. 16) is also observed to be applicable for the branched pipe system. Note that $q_{r_3\,max}$ and $q_{r_3\,min}$ correspond to $\Delta\omega_{r_3}$. From Fig. 10, $q_{r_3\,max} = 0.2703$ and $q_{r_3\,min} = 0.1323$. Substituting these values in Eq. 16, the leak discharge is obtained as $q_{olk} = 0.0069$ m^3/s and corresponds to the actual value of the leak in Fig. 9.

Discussion of results

Results show that the lowest individual leak discharge that can be detected by this method is 0.5% of the mean discharge. For the range of parameters investigated in this study, a leak at any location along the pipeline can easily be detected. For normal values of friction factors ranging from 0.01 to 0.025 encountered in real-life projects, the results show that this method is applicable for detecting and locating leaks.

Different boundary conditions at the downstream end of the branched pipe were investigated for the branched pipe system. These boundaries include: valve with constant gate discharging into atmosphere, constant-head reservoir and dead end. The results shown in Fig. 10 are for the dead end case. It is observed that this method is applicable for all the investigated cases.

Effect of approximation in the FFT method

It was shown earlier that for computational purposes, a continuous Fourier transform is modified to get a discrete Fourier transform. The resulting discrete Fourier transform that is used for computation is an approximation to the continuous transform pair. This approximation results in a slightly rough curve in the frequency response diagrams as compared to the analysis that was done in the frequency domain in the authors' previous study. Some roughness and bumps can be noticed in the curves of Figs. 4, 6, 8, and 10. These are the results of this approximation procedure and should not be considered as indicating some small leaks. Furthermore, this approximation causes some small shifts in the peaks as seen in Fig. 10. As far as the examples studied in this work are concerned, these shifts did not affect the location of the leak mainly because $\Delta\omega_r$ is measured from the peaks of the "leak case" frequency response diagram. There might be some cases where leak location could be affected by having a slightly different $\Delta\omega_r$ thus causing some uncertainties, although this situation seems remote.

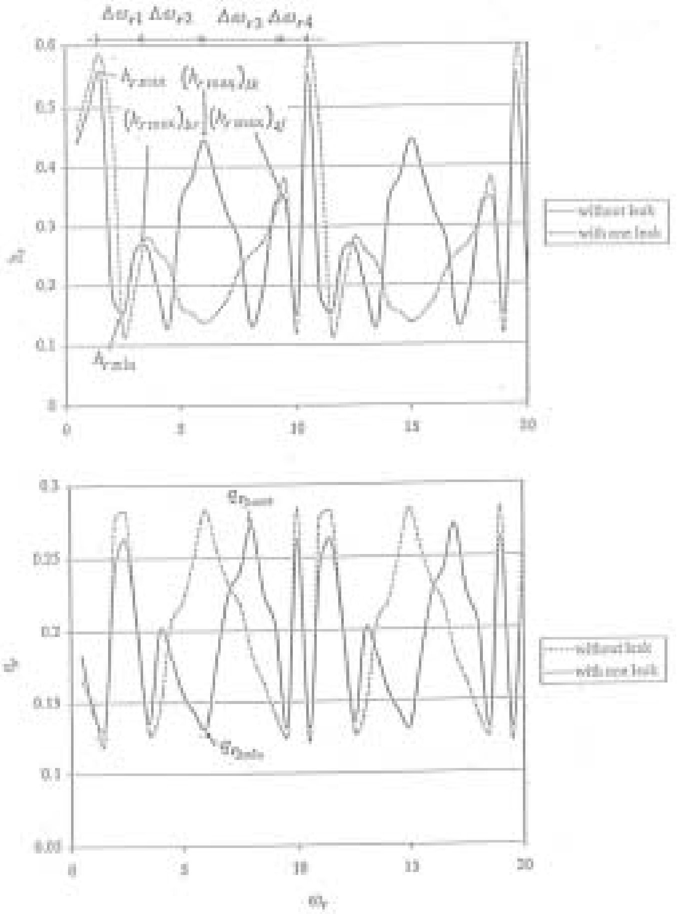

Fig. 10 Frequency Response for the Branched Pipe System Shown in Fig. 9.

Application to real-life pipe systems

The method presented herein can be used to detect leaks in real-life pipe systems. For this application, transient conditions are produced and recorded in the system by changing the settings of control equipment in the system. This could be a valve, pump, turbine, etc. The following example illustrates the procedure.

A valve located at the downstream end of a pipeline may be opened or closed gradually to produce transient flow. The operation time for opening and closing the valve should not be less than $\Sigma 4L_i/a_i$ to get the proper complete response for the entire system. The pressure head near the valve are recorded with respect to time by means of a pressure transducer. The discharge may be determined utilizing a flow meter. These experimental measurements are done during only one operation of valve closure or opening.
The measurements, which are in the time domain, are transformed into the frequency domain by using the fast Fourier transform and a frequency response diagram for the system is prepared. This is referred to as the experimental frequency response diagram. The primary, or "no-leak," response of the real-life pipe system can be modeled using the method presented herein knowing the system geometry and parameters. This is referred to as the computed frequency response diagram. By comparing the computed frequency response with the experimental frequency response, the location of a leak is indicated by the existence of additional resonant pressure amplitude peaks in the experimental frequency response diagram that are lower than the resonant pressure amplitude peaks for the computed frequency response. As discussed earlier, the magnitude of the leak is determined from the maximum and minimum discharge amplitudes in the experimental frequency response diagram. The experimental frequency response diagram for a new system or a system with no leaks may be considered and utilized as the computed frequency response diagram.

Summary and Conclusions

Leak detection is necessary to avoid economic losses, ensure safety, and control environmental and health problems. The leak detection methods presently available have several limitations and can only be employed in specified situations. The method developed by the authors in their previous work requires taking measurements of pressure and discharge amplitudes at several frequencies. The period of the valve oscillations have to be varied several times and measurements taken for each variation. In this paper, pressure and discharge measurements are taken with respect to time at the valve end when the valve is opened or closed gradually during one operation. In this case there is only one valve operation required and therefore, fewer measurements are necessary. For typical friction factors of 0.01 to 0.025 encountered in real-life projects, this method may be used to detect leaks. Individual leaks of up to 0.5% of the mean discharge can be detected and located by this method. This method can be directly used to determine the location and magnitude of leaks in real-life pipe systems by comparing a computed "no-leak" frequency response diagram to an experimentally-derived frequency response diagram

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Notations

$A, A_1, A_2 \ldots A_i \ldots A_n$ cross-sectional areas of pipe
$a, a_1, a_2 \ldots a_i \ldots a_n$ wave velocities
$C_a, C_{a_i}, C_{a_{i+1}}$ variable coefficients
C_d coefficient of discharge
C_n, C_{n_i} variable coefficients
C_p, C_{p_i} variable coefficients
$D, D_1, D_2 \ldots D_i \ldots D_n$ inside diameters of pipe
$d_{lk}, d_{lk_1}, d_{lk_2}$ diameters of leak opening
f Darcy-Weisbach friction factor
g acceleration due to gravity
H instantaneous pressure head
H_A, H_B, H_P instantaneous pressure head at A, B and P
$h(t)$ pressure head waveform
$H(\omega)$ Fourier transform for the pressure head waveform
H_o mean pressure head
$H_{Pi,n+1}$ instantaneous pressure head at section n+1 of pipe i
$H_{Pi+1,n}$ instantaneous pressure head at section 1 of pipe i+i
h_r pressure head ratio
h_{rmax}, h_{rmin} maximum an minimum pressure head ratio
$(h_{rmax})_{lk}$, $(h_{rmax})_{lk1}$, $(h_{rmax})_{lk2}\ldots$ maximum pressure head ratio at leak points
$(h_{rmax})_{sj}$, $(h_{rmax})_{br}$, $(h_{rmax})_{pa}$ maximum pressure head ratio at series, and parallel pipes junctions respectively.
$L, L_1, L_2, \ldots L_i, \ldots L_n$ lengths of pipe
Q instantaneous discharge
$q(t)$ discharge waveform
$Q(\omega)$ Fourier transform for the discharge waveform
Q_A, Q_B, and Q_P instantaneous discharge at A, B and P
Q_0 mean discharge
$Qp_{i,n+1}$ instantaneous discharge at section n+1 of pipe i
$Qp_{i+1,n}$ instantaneous discharge at section 1 of pipe i+i
q_{olk}, q_{olk_1}, q_{olk_2} mean leak discharges
q_r discharge ratio
$q_{r_{min}}, q_{r_{1\,min}}, q_{r_{2\,min}} \ldots$ minimum discharge ratios
$q_{r_{max}}, q_{r_{1\,max}}, q_{r_{2\,max}} \ldots$ maximum discharge ratios
R variable coefficient
t time
T_{th} theoretical period of the pipeline
x distance
ε variable coefficient
ω frequency
ω_{th} theoretical frequency
ω_r frequency ratio
Δt change in time
Δx change in distance
$\Delta\omega_r$, $\Delta\omega_{r_1}, \Delta\omega_{r_2}$ changes in the frequency ratio