Use of resistance coefficients derived from single planes to estimate time of concentration of two-plane systems

Utilisation des coefficients de résistance de plans uniques pour estimer le temps de concentration d'un système à deux plans

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ABSTRACT
By coupling the Darcy-Weisbach friction formula with the kinematic wave time of concentration formula, a kinematic-Darcy-Weisbach time of concentration formula for a series of planes is derived. The formula is applicable to a cascade of planes, to planes of different roughnesses, to planes of different flow regimes, to planes of different soil types and infiltration rates resulting in different net rainfall intensities, to planes subject to different rainfall intensities, and to planes with a combination of all these variables. By applying the observed times of concentration to the formula for single planes, the Darcy-Weisbach resistance coefficient for a concrete surface and for an artificial grass surface are derived. Based on the derived resistance coefficients, they indicate that the flow regime on concrete was transitional and the flow regime on artificial grass was laminar. Further, by applying the derived resistance coefficients to two-plane systems (i.e. planes with combinations of concrete surface and artificial grass surface in series), the comparisons of the estimated times of concentration from the formula with the observed values show very good agreement. In one particular two-plane system, even with the occurrence of kinematic shocks, the agreement is still good. This study shows that the resistance coefficients that are derived from uniform planes can be applied to two-plane systems.

RÉSUMÉ
En couplant la formule de frottement de Darcy-Weisbach avec la formule du temps de concentration de l'onde cinématique, on obtient la formule du temps de concentration cinématique-Dracy-Weisbach pour une série de plans. La formule est applicable, à une cascade de plans, à des plans de rugosités différentes, à des plans de différents régimes d'écoulement, à des plans de différents types de sols et taux d'infiltration conduisant à différentes intensités nettes de pluie, à des plans soumis à différentes pluviométries, et enfin à des plans combinant toutes ces variables. En appliquant les temps de concentrations observés à la formule donnée pour les plans uniques, on obtient le coefficient de résistance pour une surface en béton et pour une surface en gazon artificiel. D'après les coefficients de résistance obtenus, il est apparu que l'écoulement sur le béton était en régime de transition laminaire/turbulent, et que sur le gazon artificiel il était laminaire. Ensuite, en appliquant les coefficients de résistance obtenus pour les systèmes à deux plans (i.e. des plans combinant en série surface bétonnée et surface de gazon artificiel), les comparaisons des temps donnés par la formule sont en très bon accord avec les valeurs observées. Dans un système particulier à deux plans, l'accord est encore bon, même en présence de chocs cinématiques. Cette étude montre que les coefficients de résistance obtenus pour des plans uniformes peuvent être appliqués à des systèmes à deux plans.

1 Introduction

To estimate the time of concentration of an overland plane, the kinematic wave time of concentration formula for a single plane has been recommended (Institution of Engineers, Australia, 1987; American Society of Civil Engineers and Water Environment Federation, 1992). However, as the physical characteristics of an overland plane may not be uniform, the single plane time of concentration formula has been extended; firstly to a plane with a constant upstream inflow (Wong, 1995, 1996a), and secondly to a series of overland planes (Wong, 1996b). Of all the parameters contained in these formulae, the resistance coefficient of the overland surface is the most difficult to quantify (Chen and Wong, 1990). This difficulty is exacerbated if the planes are of non-uniform characteristics.

In order to explore the possibility of overcoming the above difficulty, experiments using simulated rainfall were conducted. The resulting runoff hydrographs from a plane with concrete surface, a plane with artificial grass surface, and from two-plane systems (i.e. planes with combinations of concrete surface and artificial grass surface in series) were recorded. From the observed hydrographs, the times of concentration were derived. Based on the observed times of concentration for homogeneous planes and applying them to the single plane formula, the resistance coefficients for the concrete surface and for the artificial grass surface were evaluated. These evaluated resistance coefficients were then applied to the formula for two-plane systems. The estimated times of concentration from the formula were compared with the experimental values.

2 Time of concentration formulae

Derived from the kinematic wave equations for flow over a plane with negligible backwater effect, the time of concentration for a series of planes is given by (Wong, 1996b):

$$t_c = \sum_{j=1}^{N} \frac{L_j}{\alpha_j^{1/\beta_j}} \left\{ \frac{\left[\sum_{r=1}^{j} (i_r L_r)\right]^{1/\beta_j} - \left[\sum_{r=1}^{j-1} (i_r L_r)\right]^{1/\beta_j}}{\left[\sum_{r=1}^{j} (i_r L_r)\right] - \left[\sum_{r=1}^{j-1} (i_r L_r)\right]} \right\} \qquad (1)$$

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where L is the length of an overland plane, i is the net rainfall intensity, N is the number of planes in series, j is the jth plane under consideration in the direction of flow for calculating the time of concentration, r is the rth plane under consideration in the direction of flow for calculating the equilibrium inflow and outflow of the jth plane, and α and β are the parameters relating the discharge per unit width of the plane, q, to the flow depth, y, as follows:

$$q = \alpha y^{\beta} \tag{2}$$

Equation (1) is applicable to a cascade of planes, to planes of different roughnesses, to planes of different flow regimes, to planes of different soil types and infiltration rates resulting in different net rainfall intensities, to planes subject to different rainfall intensities, and to planes with a combination of all these variables. In the application of (1), each plane should be of uniform slope and roughness, with one type of flow regime of constant temperature, and subject to uniform net rainfall.

In order to cater for planes with different flow regimes, the Darcy-Weisbach friction formula is used to express the discharge-depth relationship as defined by (2). The parameters α and β are:

$$\alpha = \left(\frac{8gS}{C\nu^{k}}\right)^{1/(2-k)} \tag{3}$$

$$\beta = 3/(2-k) \tag{4}$$

where g is the acceleration due to gravity, S is the slope of the overland plane in the direction of flow, ν is the kinematic viscosity of water, and C and k are the parameters relating the Darcy-Weisbach resistance coefficient, f, to the Reynolds number, R, as follows:

$$f = C/R^{k} \tag{5}$$

As the outflow from the series of planes varies with time, in order to account for the variable flow resistance due to varying discharge in (5), the Reynolds number, R, is related to the instantaneous outflow, q, as follows:

$$R = q/\nu \tag{6}$$

Substituting (3) and (4) into (1) gives:

$$t_c = \sum_{j=1}^{N} L_j \left[\frac{0.21\left(3.6\times10^{6}\nu\right)^{k_j} C_j}{S_j}\right]^{1/3} \times \left\{ \frac{\left[\sum_{r=1}^{j}\left(i_r L_r\right)\right]^{\frac{2-k_j}{3}} - \left[\sum_{r=1}^{j-1}\left(i_r L_r\right)\right]^{\frac{2-k_j}{3}}}{\left[\sum_{r=1}^{j}\left(i_r L_r\right)\right] - \left[\sum_{r=1}^{j-1}\left(i_r L_r\right)\right]} \right\} \tag{7}$$

The units are min for t_c, m for L, m^2 s^{-1} for ν, m m^{-1} for S, and mm h^{-1} for i. Earlier investigations show that k = 1 for laminar flow, and k = 0 for turbulent flow (Chow, 1959; Chow et al, 1988). For the intermediate value of k (i.e. $0 < k < 1$), the flow can be classified as transitional. Thus, (7) can be applied to a series of planes with different flow regimes.

If there are two planes in series, (7) reduces to:

$$t_c = \left[\frac{0.21\left(3.6\times10^{6}\nu\right)^{k_u} C_u}{S_u}\right]^{1/3} \left[\frac{\left(i_u L_u\right)^{\frac{2-k_u}{3}}}{i_u}\right] + \left[\frac{0.21\left(3.6\times10^{6}\nu\right)^{k_d} C_d}{S_d}\right]^{1/3} \left[\frac{\left(i_u L_u + i_d L_d\right)^{\frac{2-k_d}{3}} - \left(i_u L_u\right)^{\frac{2-k_d}{3}}}{i_d}\right] \tag{8}$$

where subscript u denotes the properties that relate to the upstream plane, and subscript d denotes the properties that relate to the downstream plane. If the net rainfall intensity is same for both planes, (8) reduces to:

$$t_c = \left[\frac{0.21\left(3.6\times10^{6}\nu\right)^{k_u} C_u L_u^{\,2-k_u}}{S_u i^{1+k}}\right]^{1/3} + \left[\frac{0.21\left(3.6\times10^{6}\nu\right)^{k_d} C_d}{S_d i^{1+k}}\right]^{1/3} \left[\left(L_u + L_d\right)^{\frac{2-k_d}{3}} - L_u^{\frac{2-k_d}{3}}\right] \tag{9}$$

If there is only one plane, (7), (8) and (9) all reduce to (Chen and Wong, 1993; Wong, 1994; Wong and Chen, 1997):

$$t_c = \left[\frac{0.21\left(3.6\times10^{6}\nu\right)^{k} C L^{2-k}}{S i^{1+k}}\right]^{1/3} \tag{10}$$

3 Experimental setup

In this study, a rainfall-runoff facility has been set up at the Nanyang Technological University (NTU) comprising a rainfall simulator, an outdoor experimental plot, and instrumentation for monitoring runoff.

3.1 Rainfall simulator

The rainfall simulator consists a steel frame, spray pipes and sprinklers. The steel frame (27.1 m long by 2.75 m wide by 4 m high) consists a upper frame, a lower frame, and four columns. The upper frame and the columns form a rigid structure. The lower frame is connected to the upper frame by four cables and can be lifted to different heights. Spray pipes and sprinklers are carried by the lower frame. The entire steel frame rests on rails such that the frame can be positioned over a desired testing bay. There are a total of twelve spray pipes, which are connected to a manifold, and the water pressure within the manifold is monitored by a pressure gauge. At the head of each pipe, a valve is installed to control the opening and closing of each line. The spray pipes are placed centrally within the steel frame. Each pipe has 23 sprinklers; the simulator has a total of 276 sprinklers. In order to ensure uniform outflow from each sprinkler on each pipe, pressure regulators are installed at the head of each line and also, at each sprinkler. Swivel spreaders are fixed to the sprinklers to enhance the uniformity of the water spray. The sprinklers are fixed to the spray pipes pointing downwards, at a spacing of 1.2 m in the longitudinal direction and 0.06 m in the transverse direction. In order to generate uniform rainfall for a range of rainfall intensities over the testing bay, the spray pipes and the sprinklers are divided into four sets. Each set, consists of three pipes and 69 sprinklers, can generate one nominal rainfall intensity. By various combinations of the four nominal intensities, a maximum and a minimum rainfall intensities can be simulated in addition to numerous intermediate intensities. Water is supplied to the rainfall simulator from a sump (lower portion of the collection chamber) by means of a centrifugal pump. After discharging from the rainfall simulator, the water returns to the sump via the testing bay. To minimise the wind effect on the simulated rainfall, the sprinklers are se at 1.7 m above and parallel to the test surface, and canvas sheets are hung from the lower frame surrounding the testing bay. For a test area of 25 m by 1 m, the simulator can generate intensities ranging from 25 to 400 mm h^{-1}. The uniformity of the simulated rainfall is assessed by means of the Christiansen uniformity coefficient (Christiansen, 1941; Hall, 1969). With this coefficient, absolute uniformity gives a value of 1.0. For the simulated rainfall, the coefficient is about 0.8.

3.2 Experimental plot

The plot consists of four testing bays and one collection chamber. The entire plot is made of reinforced concrete. The dimensions of each testing bay are 25 m long by 1 m wide. In this study, the bay with a 2% slope was used. Figure 1 shows a cross section of the rainfall simulator, the testing bay and the collection chamber.

3.3 Instrumentation

The instrumentation consists a flowmeter, weigh tanks, and a data logger. A laboratory thermometer was also used to measure the temperature of water. An electromagnetic flowmeter is installed between the pump and the manifold of the rainfall simulator. The flowmeter was first used to calibrate the weigh tanks under the steady state condition. During a hydrograph run, the flowmeter was used to monitor the steadiness of the water supply.

The weigh tank comprises a rectangular flow measurement tank (1.5 m long by 0.5 m wide by 0.5 m high) with a 20 mm wide rectangular notch at the outlet, and a weigh balance. The balance monitors the combined weight of the tank, and the water inside the tank, and sends a voltage signal to the data logger. By calibration, the voltage is related to the outflow rate through the measurement tank.

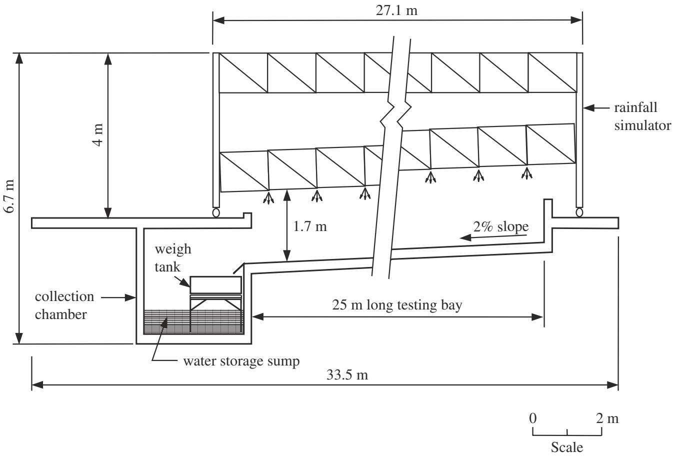

Fig. 1 Cross-section of the rainfall simulator, the testing bay and the collection chamber

4 Experimental procedure and test runs

For each hydrograph run, the rainfall simulator was set at the desired rainfall intensity. To ensure consistency in the test runs, the test surface was prepared for the saturated dry condition as recommended by Morgali (1970), i.e. no flowing water on the plane nor dryness which caused water to be absorbed by the plane. The data logging program was first started, and followed by the water supply system. A recording interval of 10-second was used. After the surface runoff has reached the equilibrium condition, the run would terminate. The temperature of the water was taken at the end of each run.

A total of two single planes and three two-plane systems were tested. For the single planes, the surfaces were concrete and artificial grass. For the two-plane systems, the surfaces comprise of various combinations of concrete and artificial grass in series. For all the cases tested, the rainfall can be considered uniform. As such, the net rainfall intensities were derived from the equilibrium outflows. Table 1 contains a summary of the test conditions.

5 Resistance coefficient

For the purpose of evaluating the resistance coefficient of an overland surface from the observed times of concentration on a single plane, the Darcy-Weisbach resistance coefficient and the Reynolds number relationship in (5) is defined as:

$$f_L = \frac{C}{R_L^{\,k}} \tag{11}$$

where f_L is the Darcy-Weisbach resistance coefficient related to the Reynolds number at the end of the plane at equilibrium R_L. The Reynolds number, R_L, is related to the rainfall intensity and the length of the plane as follows:

$$R_L = \frac{iL}{3.6\times10^6 \nu} \tag{12}$$

Substituting (11) and (12) into (10) gives (Chen and Wong, 1990, 1993; Wong, 1994):

Table 1 Summary of test conditions for each plane system

Plane System	Length and Type of Surface	Number of Runs	Net Rainfall Intensities (mm h^{-1})	Temperature of Water (^oC)
Single Plane	25 m concrete	18	72 - 322	26
Single Plane	25 m artificial grass	11	27 - 387	27
Two-plane System A	12.5 m artificial grass (u/s) - 12.5 m concrete (d/s)	6	72 - 330	24
Two-plane System B	6.25 m artificial grass (u/s) - 18.75 m concrete (d/s)	6	96 - 204	25
Two-plane System C	18.75 m concrete (u/s) - 6.25 m artificial grass (d/s)	8	71 - 333	25

$$t_c = \left[\frac{0.21L^2 f_L}{Si}\right]^{1/3} \quad (13)$$

As (10) is derived based on the instantaneous resistance coefficient and (13) is derived from (10), (13) in fact also accounts for the instantaneous resistance coefficient. The relationship between f and f_L is elaborated in Wong (1994). With the observed times of concentration from a single plane, the Darcy-Weisbach resistance coefficient of the overland surface can be evaluated from (13), which is related to the Reynolds number as defined in (12).

6 Results and analyses

As the recorded hydrograph was the outflow hydrograph from the measurement tank, the centered explicit reverse routing scheme (Wu et al, 1978; Zoppou, 1999) was used to account for the change in storage in the tank. The inflow hydrograph to the tank was then derived from the outflow hydrograph. The former was taken as the outflow hydrograph from the testing bay. Based on this outflow hydrograph, the time of concentration was estimated. As the flow approached equilibrium asymptotically, the time of concentration was defined as the commencement of flow to the time when the flow reached 97% of the equilibrium flow. This definition is consistent with Chow's (1959) interpretation of Izzard's (1946) results.

6.1 Resistance coefficients

By applying the observed times of concentration on single planes to (13), the Darcy-Weisbach resistance coefficients for the concrete surface and for the artificial grass surface were evaluated. The corresponding Reynolds numbers were evaluated from (12). Figure 2 shows a plot of the evaluated Darcy-Weisbach resistance coefficients against Reynolds numbers for the two surfaces. Further, based on experimental data of U.S. Army Corps of Engineers (1954), Yu and McNown (1964) also evaluated the Darcy-Weisbach resistance coefficient for a concrete surface. Their evaluations were based on the flow depths of the equilibrium profiles. Their results are superimposed on to Fig. 2. A comparison of the two sets of results for concrete surface show that the values evaluated in this study are higher. The differences can be attributed to the two methods. As mentioned in Section 2, the time of concentration method actually accounts for the variable flow resistance based on the instantaneous outflow. On the other hand, the flow depth method accounts for flow resistance under the equilibrium condition only. As the equilibrium flow is also the maximum outflow, the time of concentration method is expected to produce higher values of resistance coefficient.

For the purpose of getting accurate time of concentration estimates, only the resistance coefficient evaluated in this study are used in the subsequent analyses. Equation (5) is fitted to the evaluated resistance coefficients, the respective equation for the concrete surface and for the artificial grass surface are:

$$f = 4/R^{0.5} \quad (14)$$

$$f = 5000/R \quad (15)$$

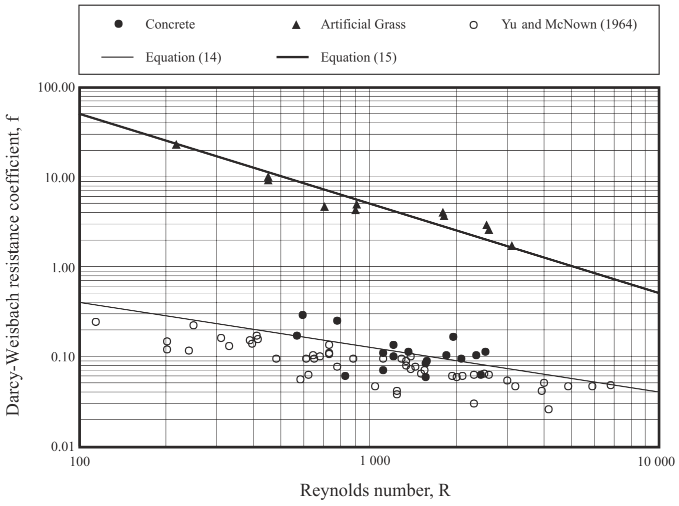

Fig. 2 Darcy-Weisbach resistance coefficients derived from single-planes for concrete surface and artificial grass surface

Based on (14) and (15), the values of C and k for the concrete surface are 4 and 0.5, and for the artificial grass surface are 5000 and 1.0. Further, the values of k in (14) and (15) indicate that the flow regime on concrete was transitional, and the flow regime on artificial grass was laminar.

6.2 Time of concentration of single planes

By substituting $\nu = 0.874 \times 10^{-6}$ m^2 s^{-1}, k = 0.5, C = 4, L = 25 m, S = 0.02 into (10), the time of concentration for the plane with concrete surface only is:

$$t_c = 21.0 i^{-1/2} \tag{16}$$

Similarly, by substituting $\nu = 0.856 \times 10^{-6}$ m^2 s^{-1}, k = 1.0, C = 5000, L = 25 m, S = 0.02 into (10), the time of concentration for the plane with artificial grass surface only is:

$$t_c = 159.3 i^{-2/3} \tag{17}$$

Figure 3 shows a comparison of times of concentration from (16) and (17) with the observed times of concentration. The good agreement between estimated values from (16) and (17) with the observed values is expected as the resistance coefficients were derived from the observed times of concentration.

6.3 Time of concentration of two-plane systems

In order to investigate whether the resistance coefficients derived from single planes are suitable for use in two-plane systems, the derived resistance coefficients are substituted into (9). By substituting $\nu = 0.915 \times 10^{-6}$ m^2 s^{-1}, $k_u = 1.0$, $C_u = 5000$, $L_u = 12.5$ m, $S_u = 0.02$, $k_d = 0.5$, $C_d = 4$, $L_d = 12.5$ m, $S_d = 0.02$ into (9), the time of concentration for the two-plane System A is:

$$t_c = 129.3 i^{-2/3} + 6.21 i^{-1/2} \tag{18}$$

Similarly, by substituting $\nu = 0.893 \times 10^{-6}$ m^2 s^{-1}, $k_u = 1.0$, $C_u = 5000$, $L_u = 6.25$ m, $S_u = 0.02$, $k_d = 0.5$, $C_d = 4$, $L_d = 18.75$ m, $S_d = 0.02$ into (9), the time of concentration for the two-plane System B is:

$$t_c = 101.8 i^{-2/3} + 10.6 i^{-1/2} \tag{19}$$

Further, by substituting $\nu = 0.893 \times 10^{-6}$ m^2 s^{-1}, $k_u = 0.5$, $C_u = 4$, $L_u = 18.75$ m, $S_u = 0.02$, $k_d = 1.0$, $C_d = 5000$, $L_d = 6.25$ m, $S_d = 0.02$ into (9), the time of concentration for the two-plane System C is:

$$t_c = 18.3 i^{-1/2} + 14.8 i^{-2/3} \tag{20}$$

Figure 4 shows comparisons of times of concentration from (18) and (20) with the observed times of concentration. Bearing in mind that the lines on Fig. 4 are obtained independently using the kinematic wave time of concentration formula (9) and the evaluated resistance coefficients from the single planes, the comparisons for all three systems are really very good. In fact, the comparisons are better for Systems A and B and poorer for System C. This can be attributed to the occurrence of kinematic shocks in System C. Nevertheless, the occurrence of kinematic shocks did not lead to serious error.

7 Conclusions

By coupling the Darcy-Weisbach friction formula with the kinematic wave time of concentration formula, a kinematic-Darcy-Weisbach time of concentration formula for a series of planes (7) is derived. This formula is applied to a two-plane system subject to uniform net rainfall intensity (9), and to a single plane (10). By applying the observed times of concentration to the formula for single planes, the Darcy-Weisbach resistance coefficients for the

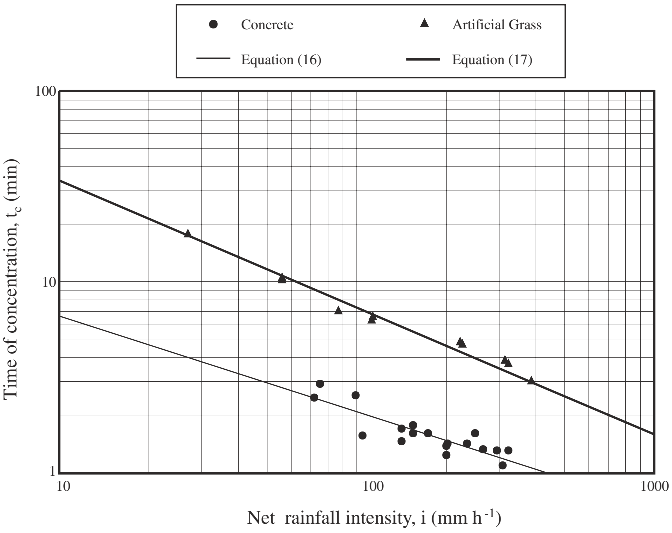

Fig. 3 Comparison of times of concentration of single planes with concrete surface and artificial grass surface

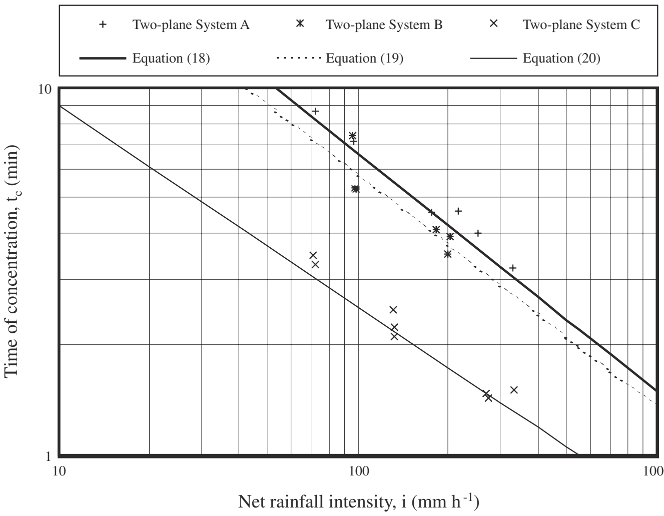

Fig. 4 Comparison of times of concentration of two-plane systems with combinations of concrete surface and artificial grass surface in series

concrete surface and for the artificial grass surface are derived. By applying the derived resistance coefficients to three different two-plane systems, each system with two types of flow regimes, the comparisons of the estimated times of concentration from the formula with the observed values show very good agreement. In one particular two-plane system, even with the occurrence of kinematic shocks, the agreement is still good. This study shows that the resistance coefficients that are derived from uniform planes can be applied to two-plane systems.

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Notations

C	parameter relating f to R
f	Darcy-Weisbach resistance coefficient
f_L	Darcy-Weisbach resistance coefficient relating to R_L
g	acceleration due to gravity
i	net rainfall intensity
k	parameter relating f to R
L	length of overland plane
N	number of planes in series
q	discharge per unit width of plane
R	Reynolds number
R_L	Reynolds number at end of plane at equilibrium
S	slope of overland plane in direction of flow
t_c	time of concentration
y	flow depth
α, β	parameters relating q to y
ν	kinematic viscosity of water

Subscripts

d	properties relating to downstream plane
j	j^{th} plane under consideration in direction of flow for calculating time of concentration
r	r^{th} plane under consideration in direction of flow for calculating upstream inflow and downstream outflow
u	properties relating to upstream plane

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