Bottom friction and time-dependent shear stress for wave-current interaction

Coefficient de frottement instantané et scisaillement de fond dans l'interaction houle-courant

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ABSTRACT

Bottom shear stresses in the wave-current interaction case are calculated using a numerical turbulent-closure model of the K-L type, where K is the turbulent kinetic energy and L is the length scale of the turbulence. Parameterized results of the friction coefficient are obtained in the case of a rough turbulent flow, as presented by Soulsby *et al.* [14], and these are here extended to the case of a smooth turbulent flow. Several comparisons with experiments and other results presented in the literature, particularly by Tanaka and Thu [19], show close agreement. A new parameterization of the time-series shear stress is proposed that includes a local friction coefficient and yields better results than the parameterization suggested by Soulsby *et al.* [14].

RÉSUMÉ

Le scisaillement de fond dans l'interaction houle-courant est calculé à l'aide du modèle numérique de fermeture turbulente de type K-L développé par Huynh Thanh [4], et les résultats obtenus pour la paramétrisation du coefficient de frottement qui ont déja été présentés pour le cas turbulent rugueux dans Soulsby *et al.* [14] sont étendus ici au cas turbulent lisse. Des comparaisons avec des expériences sont faites et donnent un meilleurs que les résultats présentés par Tanaka et Thu [19]. Une nouvelle paramétrisation du scisaillement instantané qui prend en compte un coefficient de frottement local est proposée et donne de meilleurs résultats que la paramétrisation de Soulsby *et al.* [14].

I – Introduction

Coastal morphology evolution is directly relevant to coastal management linked to human activities (e.g. ports, piers and breakwaters) and coastal erosion caused by natural phenomena (e.g. short-term storms or longer-term sea level variations). Forecasts of morphological changes are invariably dependent on the correct prediction of the sand transport rate under the action of waves and currents, which requires accurate estimation of the friction at bed level. Prediction of bottom stresses has been limited to monochromatic unidirectional waves, and ignored wave irregularity, multi-directionality, non-linearity, asymmetry and breaking. In recent years, various attempts have been made to improve the state of knowledge regarding complex wave effects on the sand-transport rate, using theoretical models.

In 1981, Tanaka and Shuto derived an implicit theoretical solution based on the hypothesis of time-independent turbulent viscosity for bottom friction under the action of a sinusoidal wave interacting with a current in a turbulent flow regime. Later, Tanaka and Shuto proposed an explicit solution that took account of whether the turbulent flow regime was rough or smooth. This solution is summarized in Section II.

During the European MAST I/G6M Coastal Morphodynamics program, comparison of several numerical models that take instantaneous turbulent viscosity into account led to a parameterization of the maximum value τ_{max} and the mean value τ_m of the shear stresses, for interaction between a sinusoidal wave and an oblique current, for a rough turbulent flow. These results were published in Soulsby *et al.* [14].

This paper will make use of Huynh Thanh and Temperville's [5] numerical model (referred to as HT91 in Soulsby *et al.* [14]), which was used in this inter-comparison. Details of this model are presented in Huynh Thanh [4]. In short, this equations system model is established with the following assumptions: (1) the thickness of the boundary layer is much smaller than the wavelength of the wave; (2) the amplitude of the wave velocity $\hat{U}_w$ is much smaller than the wave celerity. In these conditions, the momentum equations for the horizontal components of the velocity (*u, v*), and the *K* and *L* equations for the turbulent closure can be written:

$$\frac{\partial u}{\partial t} = \frac{\partial U_w}{\partial t} - \frac{1}{\rho}\frac{\partial P_c}{\partial x} + \frac{\partial}{\partial z}\left(\upsilon_t \frac{\partial u}{\partial z}\right)$$

$$\frac{\partial v}{\partial t} = \frac{\partial V_w}{\partial t} - \frac{1}{\rho}\frac{\partial P_c}{\partial y} + \frac{\partial}{\partial z}\left(\upsilon_t \frac{\partial v}{\partial z}\right)$$

$$\frac{\partial K}{\partial t} = \upsilon_t \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - \frac{\upsilon_t}{L^2}K + 1.2\frac{\partial}{\partial z}\left(\upsilon_t \frac{\partial K}{\partial z}\right)$$

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$$\frac{\partial L}{\partial t} = 0.175\frac{\upsilon_t}{K}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]L + 0.075\sqrt{2K}$$

$$+1.2\frac{\partial}{\partial z}\left(\upsilon_t\frac{\partial L}{\partial z}\right) - \frac{0.375\sqrt{2}}{\sqrt{K}}\left[\frac{\partial\left(\sqrt{K}L\right)}{\partial z}\right]^2$$

where (U_w, V_w) are the horizontal components of the wave velocity, P_c represents the pressure due to the current, and the turbulent viscosity is obtained assuming local equilibrium of the turbulence, which allows υ_t to be put in the form:

$$\upsilon_t = \frac{\sqrt{2K}}{4}L$$

Tran Thu [21] used the same numerical model in the smooth turbulent flow study, by introducing certain modifications to the boundary conditions. Section II of this paper presents a selection of these results. Other results obtained using this model for a sinusoidal wave and different turbulent regime flows are compared with Tanaka's results and experimental data in part III-1. In part III-2, the results of the present model are compared with those obtained by Tanaka for a regular wave with a current interaction in either smooth or rough turbulent regimes. With the different figures for the rough turbulent flow, we also present results obtained through Soulsby's [14] parameterization using Fredsøe's coefficients.

Soulsby and Ockenden [13] show that the instantaneous shear stress of an irregular wave could be calculated by defining an equivalent monochromatic wave, with orbital velocity amplitude $U_w = \sqrt{2}\ U_{rms}$, period T_p, and where the direction of propagation is the mean direction of the irregular waves. In part IV, we will use the present model to show that the effect of "*the turbulence history*" could greatly influence the case of asymmetrical waves. Taking this effect into account, and following the approach of Soulsby and Ockenden [13], we propose a new formulation for calculating the time-series of shear stresses.

II – Description of Models

II.1 – Tanaka's model

Tanaka and Shuto's [17] and [18] model is based on the hypothesis that the turbulent viscosity is time-independent. In the case of interaction between a current and a sinusoidal wave whose direction forms an angle ϕ with that of the current, Tanaka and Shuto derived a theoretical solution for the friction coefficient f_{cw}, defined by the relationship:

$$\frac{\tau_{\max}}{\rho} = U_{cw}^{*\,2} = \frac{f_{cw}}{2}\hat{U}_w^{\,2}$$

where τ_{max} represents the maximum shear stress value in the wave-current interaction, U_{cw}^* is the maximum shear velocity under waves and current, and $\hat{U}_w$ represents the orbital velocity amplitude of the wave at the upper limit of the bottom boundary layer.

The theoretical solution of the friction coefficient proposed in the case of the turbulent flow, which could be either smooth or rough, is obtained by iteration. More recently, Tanaka and Thu [19] derived an explicit solution for the different flow regimes: rough turbulent flow, smooth turbulent flow and laminar flow, as well as for the transitory flow regime in the sinusoidal wave-current interaction. The friction coefficient f_{cw} is given by the following formula:

$$f_{cw} = \tilde{f}_c + 2\sqrt{\beta\tilde{f}_c f_w}\cos(\phi) + \beta f_w \qquad \tilde{f}_c = f_c\left(\frac{U_c}{\hat{U}_w}\right)^2 \tag{1}$$

where the term β depends on the turbulent flow case (rough or smooth); it is defined below.

The current and the wave friction coefficients f_c and f_w are defined, respectively, by:

$$\frac{\tau_c}{\rho} = \frac{f_c}{2}U_c^{\,2} \qquad \frac{\hat{\tau}_w}{\rho} = \frac{f_w}{2}\hat{U}_w^{\,2} \tag{2}$$

where τ_c represents the bottom shear stress due to the current, $\hat{\tau}_w$ represents the maximum absolute value for the shear stress due to the wave. f_c and f_w are calculated by:

a) in the rough turbulent flow case

$$f_c = f_{c(r)} \qquad f_w = f_{w(r)}$$

$$f_{c(r)} = 2\left(\frac{k}{Ln(\frac{h}{z_0}) - 1}\right)^2 \qquad f_{w(r)} = \exp\left[-7.53 + 8.07\left(\frac{A}{z_0}\right)^{-0.1}\right] \tag{3}$$

$$\beta = \frac{1}{1+0.769\,\alpha^{0.83}}\left[1 + 0.863\ \alpha\ \exp(-1.43\ \alpha)\left(\frac{2\phi}{\pi}\right)^2\right]$$
$$\alpha = \frac{1}{\ln(\frac{h}{z_0}) - 1}\left(\frac{U_c}{\hat{U}_w}\right) \tag{4}$$

k represents von Kármán's constant (k = 0.4), z_0 the roughness length ($z_0 = K_N/30$), ϕ the angle between the wave and current directions, in radians, h the water depth, and A the wave excursion amplitude at the upper limit of the bottom boundary layer.

b) in the smooth turbulent flow case

$$f_c = f_{c(s)} \qquad f_w = f_{w(s)}$$

$$f_{c(s)} = \exp[-7.60 + 5.98\,R_c^{-0.0977}]$$
$$f_{w(s)} = \exp[-7.94 + 7.35\,R_w^{-0.0748}] \tag{5}$$

$$\beta = \frac{1 + 0.871\, R_c^{-0.0362}\, f_{c(s)}^{\;\;0.177} \left(\frac{2\phi}{\pi}\right)^{2.5}}{1 + 5.04\, R_c^{-0.0303}\, f_{c(s)}^{\;\;0.379}} \tag{6}$$

$R_c = \dfrac{U_c\, h}{\nu}$ is the current Reynolds number

$R_w = \dfrac{\hat{U}_w\, A}{\nu}$ is the wave Reynolds number

II.2 – Soulsby's model

a) Rough turbulent flow case. Huynh Thanh's model [4]

During the European MAST I/G6M Coastal Morphodynamics program, several sophisticated models were used to calculate the bottom shear stress in the interaction of a sinusoidal wave with a current, in the rough turbulent flow case. Those numerical models took the time-dependence of the turbulent viscosity into account. After comparing the results, Soulsby *et al.* [14] proposed the following parameterization:

$$Y_1 = 1 + a\, X_1^{\;m} \left(1 - X_1\right)^n \qquad y_1 = X_1 \left(1 + b\, X_1^{\;p} \left(1 - X_1\right)^q\right) \tag{7}$$

$$Y_1 = \frac{\tau_{max}}{\tau_c + \hat{\tau}_w} \qquad y_1 = \frac{\tau_m}{\tau_c + \hat{\tau}_w} \qquad X_1 = \frac{\tau_c}{\tau_c + \hat{\tau}_w} \tag{8}$$

where the coefficients a, m, n, b, p and q were given by an expression of the following form:

$$\begin{aligned} a &= a_1 + a_2 \left|\cos\phi\right|^I + (a_3 + a_4 \left|\cos\phi\right|^I)\ \log_{10}(f_{w(r)} / f_{c(r)}) \\ b &= b_1 + b_2 \left|\cos\phi\right|^J + (b_3 + b_4 \left|\cos\phi\right|^J)\ \log_{10}(f_{w(r)} / f_{c(r)}) \end{aligned} \tag{9}$$

The first expression of (9) is for the coefficients a, m and n, whereas the second one is for the coefficients b, p and q.
Table 1 gives the values of the fitting coefficients a_i, m_i, n_i, I, b_i, p_i, q_i and J for this model (referred to as HT91 by Soulsby *et al.* [14]) and Fredsøe's model (referred to as F84).

Using the present model, Huynh Thanh [4] proposed formula (10) below for the wave friction coefficient $f_{w(r)}$, in the rough turbulent flow case:

$$f_{w(r)} = c_1 \exp\left[c_2 \left(\frac{A}{K_N}\right)^{n_1}\right] \tag{10}$$

with the empirical coefficients c_1, c_2 and n_l determined by Huynh Thanh, and presented in Table 2 (formula HT_{fwr}).
Using the same boundary layer model (HT91), considering the best overall fit with a large number of the model results, in the interval $6.4*10^{-1} \leq A/K_N \leq 3.4*10^3$, we propose formula (10) with the empirical coefficients determined in the present study, listed in Table 2 as formula CT_{fwr}.

Table 2. Fitting coefficients c_1, c_2 and n_l, for model of Huynh Thanh [4] = HT_{fwr} and proposed model = CT_{fwr}

Formula \ Coefficients	a_1	a_2	n_1
HT_{fwr}	0.00278	4.65000	-0.22000
CT_{fwr}	0.00140	4.58400	-0.13400

In the case of a current alone, Huynh Thanh found that the friction coefficient $f_{c(r)}$ coincides with the value obtained by the theoretical formula (3).

b) Smooth turbulent flow case. Tran Thu's model [21]

In the course of the European MAST II/G8M Coastal Morphodynamics program, Tran Thu [21] used the present model to study the smooth turbulent flow case. In the case of a pure current, he found that the friction coefficient $f_{c(s)}$ coincides with the implicit theoretical formula:

$$\frac{1}{\sqrt{f_{c(s)}}} = 1.768\ \ln(R_c \sqrt{f_{c(s)}}) + 1.554 \tag{11}$$

which could be approximated through the explicit formula given by Tanaka and Thu [19]:

Table 1. Fitting coefficients a_i, m_i, n_i, I, b_i, p_i, q_i and J, for models of Fredsøe [3] = F84; Huynh-Thanh and Temperville [5] = HT91, and Tran Thu [21] = TT95

	a_1	a_2	a_3	a_4	m_1	m_2	m_3	m_4	n_1	n_2	n_3	n_4	I
HT91	-0.070	1.870	-0.340	-0.120	0.720	-0.330	0.080	0.340	0.780	-0.230	0.120	-0.120	0.820
F84	-0.060	1.700	-0.290	0.290	0.670	-0.290	0.090	0.420	0.750	-0.270	0.110	-0.020	0.800
TT95	0.711	0.542	-0.167	0.784	0.238	-0.034	2.711	-1.091	0.368	-0.011	0.266	0.268	0.800
	b_1	b_2	b_3	b_4	p_1	p_2	p_3	p_4	q_1	q_2	q_3	q_4	J
HT91	0.270	0.510	-0.100	-0.240	-0.750	0.130	0.120	0.020	0.890	0.400	0.500	-0.280	2.700
F84	0.290	0.550	-0.100	-0.140	-0.770	0.100	0.270	0.140	0.910	0.250	0.500	0.450	3.000
TT95	0.311	0.347	-0.205	0.061	-0.570	-0.070	0.036	0.522	0.809	0.491	0.475	1.326	3.800

$$f_{c(s)} = \exp\left[-7.60 + 5.98\ R_c^{-0.0977}\right] \quad (12)$$

In the case of a wave alone, Tran Thu found that the friction coefficient $f_{w(s)}$ could be parameterized by the following formula:

$$\frac{1.16}{4\sqrt{f_{w(s)}}} + \log_{10}\left(\frac{1}{4\sqrt{f_{w(s)}}}\right) = \log_{10}(R_w) - 1.16 \quad (13)$$

which may be approximated through the explicit one:

$$f_{w(s)} = 0.39\ R_w^{\left(-0.44\ +\ 0.02\ \log_{10} R_w\right)} \quad (14)$$

In expressions (8), τ_{max} and τ_m are non-linear maximum bottom shear stress and mean bottom shear stress, respectively, and $\tau_c + \hat{\tau}_w$ could be interpreted as the maximum superposition stress without non-linear effects of a unidirectional wave-current case. As a consequence of non-linear effects, we can state: $Y > 1$ and $y > x$ in $0 < x < 1$, and expressions (7) correctly reflect these properties in a unidirectional wave-current interaction. However, in a general oblique wave-current interaction case, conservation of these properties requires the use of a vectorial addition $\vec{\tau}_c + \vec{\tau}_w$, so $\left|\vec{\tau}_c + \vec{\tau}_w\right|$, instead of the arithmetical addition $\tau_c + \hat{\tau}_w$ to continue using expressions (7) and (9). So, as suggested by Tran Thu, we propose a representation of Y_2 and y_2 as a function of X_2 defined by:

$$Y_2 = \frac{\tau_{max}}{\left|\vec{\tau}_c + \vec{\tau}_w\right|} \quad y_2 = \frac{\tau_m}{\left|\vec{\tau}_c + \vec{\tau}_w\right|} \quad X_2 = \frac{\tau_c}{\left|\vec{\tau}_c + \vec{\tau}_w\right|} \quad (15)$$

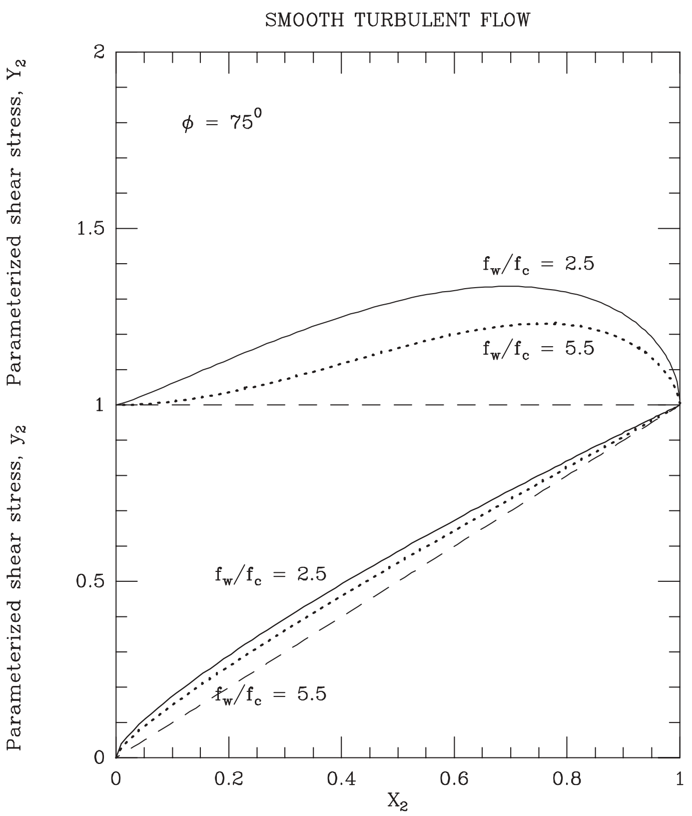

Fig. 1. Parameterized curves $Y(X_2)$ and $y(X_2)$, as suggested by Tran Thu [21]: formulations (16)

Contrary to the results obtained by Soulsby's formulation, where the curve representing Y_1 as a function of X_1 could cross the X_1 axis between the values 0 and 1, figure 1 shows that this phenomenon disappears, allowing the proposal of a parameterization of the form:

$$Y_2 = 1 + a\,X_2^{\,m}(1 - X_2)^n \qquad y_2 = X_2\left(1 + b\,X_2^{\,p}(1 - X_2)^q\right) \quad (16)$$

where the fitting coefficients to obtain the values of *a, m, n, I* and *b, p, q, J* for the model of Tran Thu (referred to as TT95) are given in Table 1.

It is therefore easy to establish the relationship between (X_1, Y_1) and (X_2, Y_2):

$$\frac{X_1}{X_2} = \frac{Y_1}{Y_2} = \sqrt{1 - 2X_1(1 - X_1)(1 - \cos\phi)} \quad (17)$$

III – Comparisons between different models

III.1 – Sinusoidal wave alone

a) Rough turbulent flow case

For values of the wave friction coefficient, $f_{w(r)}$, we propose formula (10) with CT_{fwr} coefficients (Table 2); Tanaka and Thu [19] suggested formula (3), Swart [16] formula (18) and Soulsby *et al.* [14] formula (19):

$$f_{w(r)} = 0.00251\ \exp\left[5.21\left(\frac{A}{K_N}\right)^{-0.19}\right] \quad (18)$$

$$f_{w(r)} = 1.39\left(\frac{A}{z_0}\right)^{-0.52} \quad (19)$$

A comparison between formulae (3), (10) with HT_{fwr} and CT_{fwr} coefficients, (18) and (19) is shown in figure 2.

The same figure also gives the experimental measurements of Sleath [11], Kamphuis [9], Jensen *et al.* [6], Sumer *et al.* [15] and Jonsson and Carlsen [8]. According to Sleath [12], bottom shear stress may be split into two components:

$$\hat{\tau}_{wp} = \hat{\tau}_w + \hat{\tau}_p$$

The shear stress in the fluid, $\hat{\tau}_w$, is taken into account by the model, but the value of $\hat{\tau}_p$, due to the mean pressure gradient acting on the bed roughness, is not. Using Sleath's experiments, it can be seen that a global friction coefficient may be split into the following two components:

$$f_{wp} = f_w + f_p$$

where f_w represents the friction coefficient obtained by the K-L model, and f_p represents the pressure gradient contribution. Assuming $K_N = 2.5\ D_{50}$, Sleath [12] presented the formula:

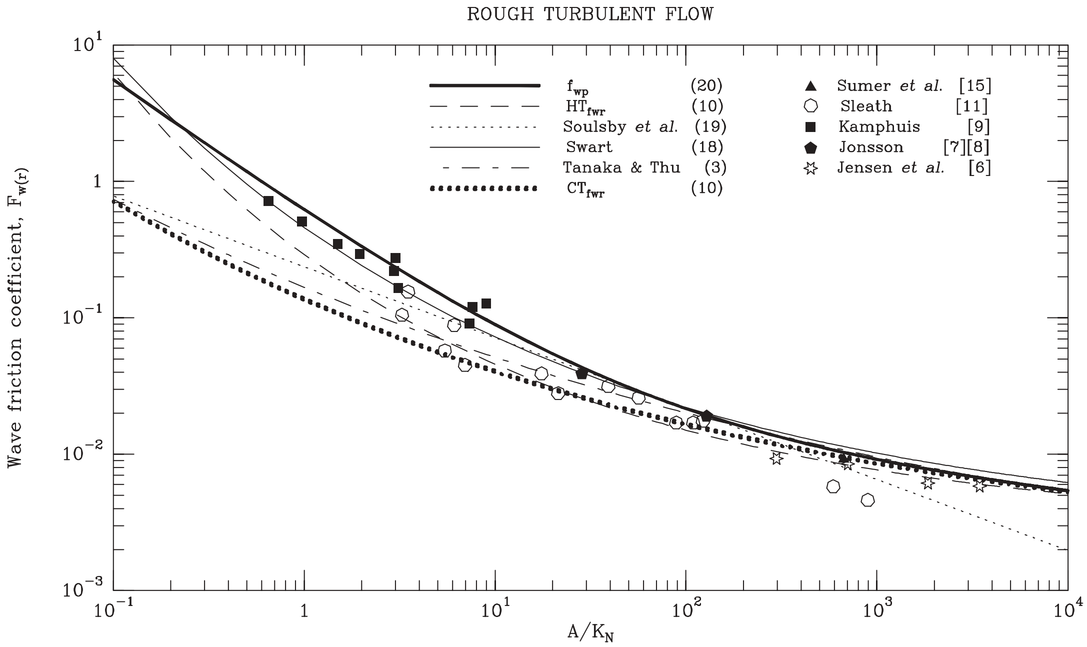

Fig. 2. Parameterization for the wave friction coefficient, f_w, in the rough turbulent regime

$$f_p = 0.48 \left(\frac{A}{K_N}\right)^{-1}$$

The pressure gradient was not taken into account in experiments conducted by Sleath, Sumer, Jensen and Jonsson. Therefore, results of their experimental data are compared with our proposed model (10) with CT_{fwr} coefficients. Apart from some of Sleath's experiments, particularly those with values of A/K_N around 3.5 and those in the range of 580 to 900, all other cases show a close agreement. Discrepancies are explained as a consequence of some of Sleath's experiments being in the smooth-laminar transition regime. The pressure gradient is taken into account in Kamphuis' experiments, so this data should be compared with values for the following expression:

$$f_{wp} = f_w + f_p = 0.0014\ \exp\left[4.584 \left(\frac{A}{K_N}\right)^{-0.134}\right] + 0.48 \left(\frac{A}{K_N}\right)^{-1} \qquad (20)$$

which are also presented in figure 2. Results of the proposed model are closer to those of Kamphuis than those of Tanaka. It can be seen that Swart curve is also in good agreement with Kamphuis experimental values of *f*. Finally, it is a matter of fact that for values of $A/K_N > 100$, the f_p term is negligible and expression (10) with CT_{fwr} coefficients (Table 2) is in close agreement with results.

b) Smooth turbulent flow case

For values of the wave friction coefficient, $f_{w(s)}$, Tran Thu and Temperville [20] proposed formula (14), Tanaka and Thu [18] suggested formula (5), and Fredsøe [3] formula (21):

$$f_{w(s)} = 0.035\ R_w^{-0.16} \qquad (21)$$

A comparison of formulae (5), (14), and (21) is shown in figure 3. The same figure presents results for the transition between laminar and smooth turbulent flows, obtained using formula (23), with $f_2 = 1$.

Experimental results of Arnskov *et al.* [2], with Reynolds numbers in the interval $10^3 – 2*10^4$, are presented in this figure. Those results are in agreement with the theoretical findings for the laminar flow case. The experimental data of Sumer *et al.* [15] and Jensen *et al.* [6] is close to the transition curve between the laminar and smooth turbulent regimes. The experiments of both Jensen and Sleath with Reynolds numbers $1.6*10^5$, $2.9*10^5$, $1.13*10^5$ and $2.52*10^5$, respectively, are in the transition regime.

c) General case

It is known that in the laminar case the friction coefficient is given by:

$$f_{w(l)} = \frac{2}{\sqrt{R_w}} \qquad (22)$$

Using $f_{w(l)}$, defined by (22), and the two friction coefficients defined in the case of rough turbulent flow by (3) and in that of the smooth turbulent regime by (5), Tanaka proposed a general formula that regroups all regimes, including transitory cases.

$$f_{wg} = f_2 (f_1\, f_{w(l)} + (1 - f_1) f_{w(s)}) + (1 - f_2) f_{w(r)} \qquad (23)$$

$$f_1 = \exp\left[-0.0513 \left(\frac{R_w}{R_0}\right)^{4.65}\right]$$

$$f_2 = 1.0 \qquad \text{if} \qquad R_w < R_1$$

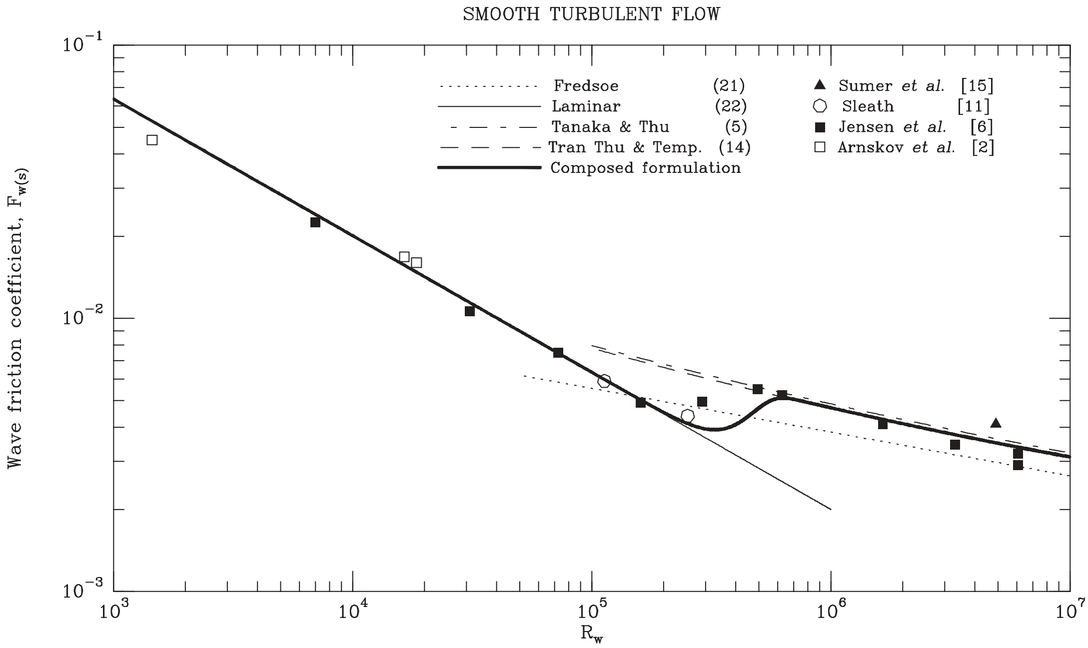

Fig. 3. Parameterization for the wave friction coefficient, f_w, in the smooth turbulent regime

$$f_2 = \exp\left[-0.0101\left(\frac{R_w}{R_1}\right)^{2.06}\right] \quad \text{if} \quad R_1 \le R_w \le R_2$$

$$f_2 = 0 \quad \text{if} \quad R_w > R_2$$

$$R_0 = 2.5*10^5 \quad R_1 = 0.501\left(\frac{A}{z_0}\right)^{1.15} \quad R_2 = 7.0\left(\frac{A}{z_0}\right)^{1.15}$$

Using $f_{w(l)}$, defined by (22), and the two friction coefficients defined (according to the present model) by (20), f_{wp}, in the case of rough turbulent flow, and by (14), $f_{w(s)}$, in the case of smooth turbulent flow, f_{wg} can be represented for all regimes adopting the same Tanaka's coefficients, f_1 and f_2. These results, and the results of Kamphuis' [9] experiments, are presented in figure 4. The results given by the proposed formulation are in better agreement with the experimental ones than those presented in Tanaka and Thu [19], particularly for weak values of the relative bed orbital excursion, A/z_0.

III.2 – Wave-current interaction case

Tanaka's results for the friction coefficient can be compared with those of the present model in the wave-current interaction case; for that, they must be expressed in Soulsby's proposed form:

$$Y_1 = 1 + a\, X_1^{\,m}\left(1 - X_1\right)^n$$

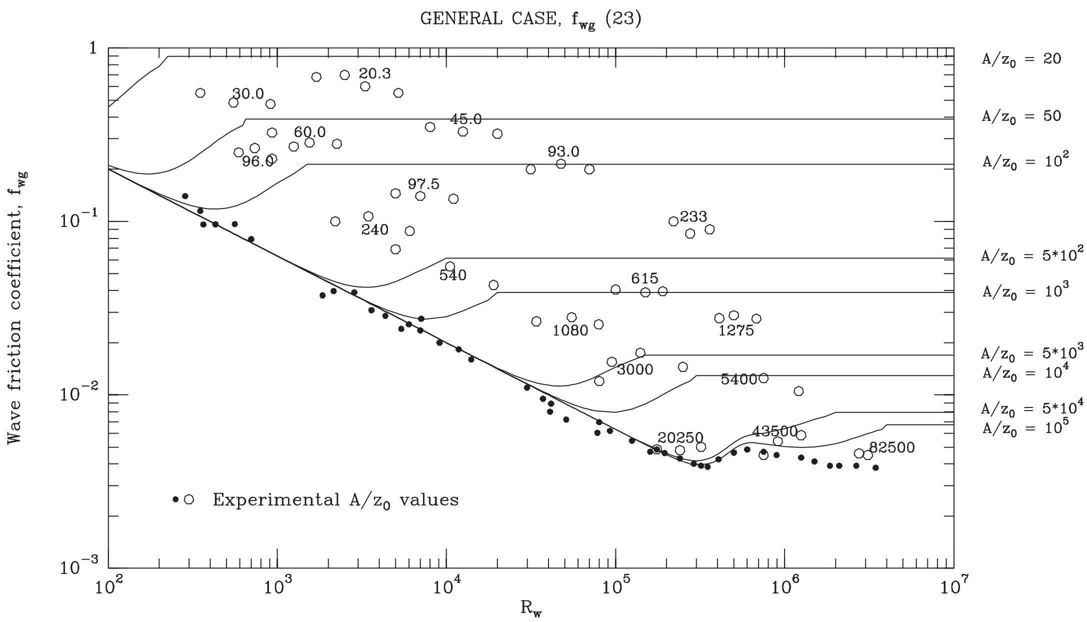

Fig. 4. Wave friction coefficient, f_w, computed from model (20), in the case of rough turbulent flow, and from model (14) in the case of smooth flow. Data from Kamphuis [9], reproduced from Tanaka and Thu [19]

$$Y_1 = \frac{\tau_{max}}{\tau_c + \hat{\tau}_w} \qquad X_1 = \frac{\tau_c}{\tau_c + \hat{\tau}_w}$$

Equations (2) and (8) make it possible to write:

$$\frac{\tau_c}{\hat{\tau}_w} = \frac{f_c}{f_w}\left(\frac{U_c}{\hat{U}_w}\right)^2 = \frac{X_1}{1 - X_1} \tag{24}$$

$$\left(\frac{U_c}{\hat{U}_w}\right) = \sqrt{\frac{X_1}{1 - X_1}}\sqrt{\frac{f_w}{f_c}} \tag{25}$$

Equation (1) may also be written:

$$\tau_{max} = \tau_c + 2\sqrt{\beta\,\tau_c\,\hat{\tau}_w}\cos\phi + \beta\,\hat{\tau}_w$$

$$Y_1 = X_1 + 2\sqrt{\beta\,X_1\left(1 - X_1\right)}\cos\phi + \beta\left(1 - X_1\right) \tag{26}$$

a) Rough turbulent regime case

Taking (25) into account, equation (4) is written:

$$\alpha = \frac{1}{\ln\left(\frac{h}{z_0}\right) - 1}\sqrt{\frac{X_1}{1 - X_1}}\sqrt{\frac{f_w}{f_c}} \tag{27}$$

In this way, through the use of coefficients β and α (equations (4)), formulation (26) depends on angle ϕ and on the relationships f_w / f_c and h / z_0, as opposed to Soulsby's formulation (7), which depends only on angle ϕ and on the relationship f_w / f_c. In practice, parameter h / z_0 is in the order of 10^4 to 10^5. Figure 5 shows the representative Y_1 curve according to the present model (HT91 coefficients), and to Tanaka's model, for $h / z_0 = 10^4$ and 10^5 in the case of angle ϕ = 0, 30 and 60 degrees, and $f_w / f_c = 5.0$. The same figure gives the results obtained with Fredsøe's (F84) coefficients.

b) Smooth turbulent regime case

Taking into account equations (1), (5) and (6), through coefficient β, formulation (26) depends on angle ϕ, the relationship f_w / f_c and the current Reynolds number R_c, as opposed to Tran Thu's formulation (15), which depends only on angle ϕ and on the relationship f_w / f_c. In practice, parameter R_c is in the order of 10^4 to 10^6. Figure 6 presents the representative *Y* curve, according to the present model (TT95 coefficients) and Tanaka's model, for $R_c = 10^4$ to 10^6, with angle ϕ = 0, 30 and 60 degrees, and $f_w / f_c = 5.0$.
Table 3 summarises the proposed parametric formulae for bottom friction, for both rough and smooth turbulent flow regimes and for current, wave and the wave-current interaction.

IV – Time-dependent shear stress

For the purpose of calculating time-dependent shear stress τ(*t*) in the case of an irregular wave whose instantaneous velocity is given by *U*(*t*), Soulsby *et al.* [14] propose calculating the value of the friction coefficient f_w for the equivalent sinusoidal wave with orbital velocity amplitude equal to $\sqrt{2}\,U_{rms}$ and period T_p. It can therefore be deduced:

$$f_w = 1.39\left(\frac{A}{z_0}\right)^{-0.52} \qquad A = \frac{\sqrt{2}\,U_{rms}}{2\,\pi}$$

where U_{rms} = root-mean-square of orbital velocities.
For a sinusoidal wave, this formulation correctly represents, in parametric form, the bottom shear stress obtained using the present K-L model, but does not take into account the phase shift between τ(*t*) and *U*(*t*). For an asymmetric wave, or an irregular wave, more important differences appear between this parametric

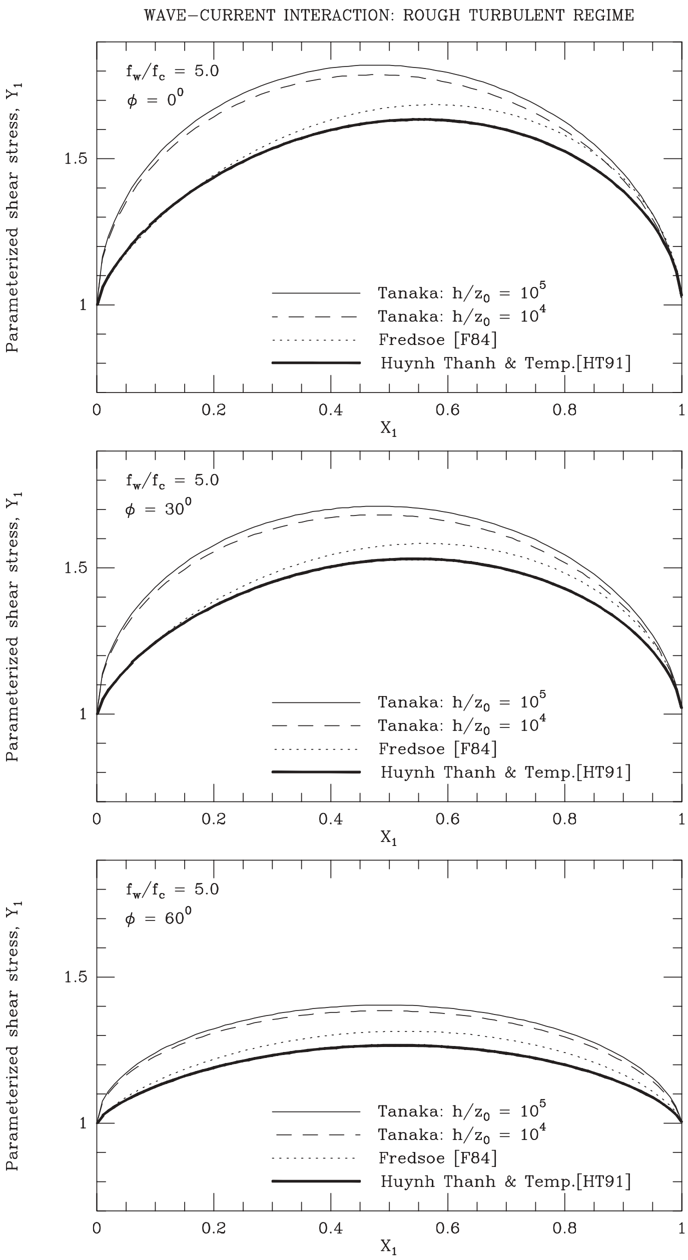

Fig. 5. Normalized maximum shear stress results for rough bottom in the wave-current interaction, computed from the models of Fredsøe [3], Huynh Thanh & Temperville [5] and Tanaka & Thu [19]

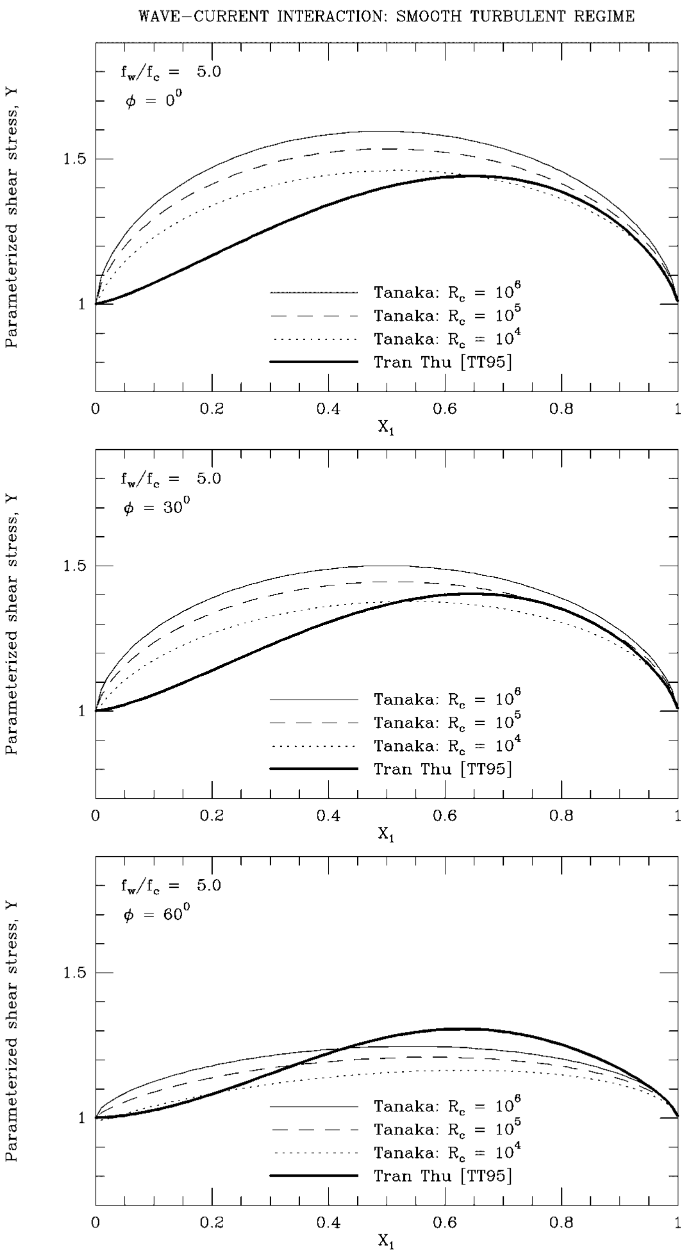

Fig. 6. Normalized maximum shear stress for smooth bottom in the wave-current interaction, computed from the models of Tanaka & Thu [19] and Tran Thu [21]

formulation and results calculated directly with the present K-L model.

To illustrate these phenomena, we consider the instantaneous velocity records presented in figure 7 for three cases: a) a sinusoidal wave, with orbital velocity amplitude = 0.225 m/s and period = 3.6 sec; b) a cnoidal wave, with a total velocity amplitude = 1.107 m/s and period = 9 sec, and c) an irregular wave obtained by the non-linear propagation of a sinusoidal wave, making use of a numerical Boussinesq-type model (Antunes do Carmo *et al.* [1]), with a period = 3.0 sec in a channel 0.30 m depth. The instantaneous bottom shear stresses $\tau(t)$ are calculated using the proposed model (10) with C_{fwr} coefficients (result 1). Results given by the K-L model (result 2) are compared both with those of the proposed model (result 1) and with those obtained by a constant friction coefficient without the phase shift (result 3).

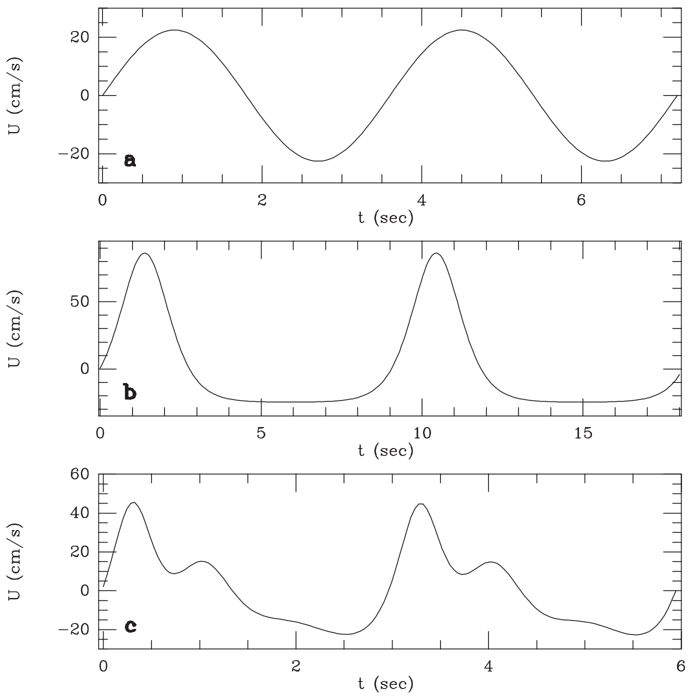

Fig. 7. Instantaneous velocity records: a – Sinusoidal wave (orbital velocity amplitude = 0.225 m/s, period = 3.6 s); b – Cnoidal wave (total velocity amplitude = 1.107 m/s, period = 9.0 s); c – Irregular wave (resulting from the non-linear propagation of a sinusoidal wave with a period = 3.0 s in a channel 0.30 m depth)

The values of the friction coefficient for a sinusoidal wave are shown in figure 8. Close agreement is evident between results 1 and 2.

Computed shear stresses for the sinusoidal wave case are presented in figure 9a. Results of the proposed model (10) with CT_{fwr} coefficients (result 1) are in close agreement with those of the K-L model. A phase error between result 3 and result 2 (K-L model) is evident.

In the cnoidal wave case, the bottom shear stress calculated by the

Table 3. Parameterized bottom friction formulae for rough and smooth turbulent flows, and for current, wave and the wave-current interaction

	Rough turbulent flow	**Smooth turbulent flow**
Current	$f_{c(r)}$ (3)	$f_{c(s)}$ (12)
Wave	$f_{w(r)}$ (10) with CT_{fwr} coefficients, or f_{wp} (20)	$f_{w(s)}$ (14)
Wave-current interaction	Parameterized Y_1 curve (7), with HT91 coefficients	Parameterized Y_2 curve (16), with TT95 coefficients

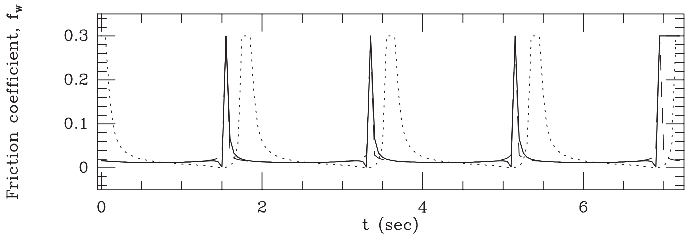

Fig. 8. Comparisons between the parameterized friction coefficient and the K-L model result for a sinusoidal wave. Proposed model (result 1: -------- ; result 3: ········) and that obtained by K-L model (result 2: ——)

numerical boundary layer model is represented in figure 9b by the continuous line.

As can be seen, for this case (figure 9b), result 1 is closer to result 2 than it is to result 3, for both phase and negative values. However, asymmetries are not reproduced and a discrepancy can be seen for the maximum value.

Several observations can be made concerning these results:

i) – The representative curve $\tau(t)$ does not present the symmetry of velocities $U(t)$. The negative values of $\tau(t)$ are more important after the main positive peak than before it. It may be assumed that a "*turbulence memory*" created for this main peak influences what happens afterwards.

ii) – If the maximum velocity value is considered to be U_1 and the minimum velocity value U_2, according to (2) it follows that:

$$\frac{\tau_2}{\tau_1} = \left(\frac{U_2}{U_1}\right)^2 = 0.08 \tag{28}$$

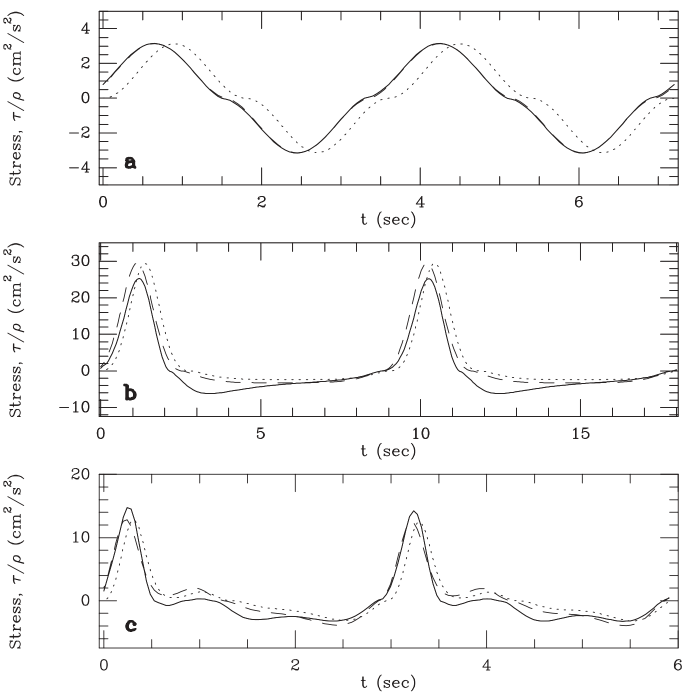

Fig. 9. Comparisons between the parameterized shear stress and the K-L model result: a) Sinusoidal wave; b) Cnoidal wave; c) Irregular wave. Proposed model (result 1: -------- ; result 3: ········) and that obtained with K-L model (result 2: ——)

Figure 9b shows that the relation $\frac{\tau_2}{\tau_1} = 0.24$ is greater than the value calculated by (28). Therefore, as in the case of a sinusoidal wave, the friction coefficient does not remain constant when velocity changes, assuming increasing values with decreasing velocity. We propose calculating a time-dependent friction coefficient by replacing the maximum velocity with the instantaneous velocity $U(t + \theta)$, which takes into account the phase shift. The coefficient $f(t)$ will accordingly be calculated using expression (10), with CT_{fwr} coefficients (Table 2), where A is given by:

$$A = \frac{\sqrt{2}\, U_{rms}\, T_p}{2\pi} \frac{U(t+\theta)}{U_{\max}} \tag{29}$$

and $\tau(t)$ is defined by:

$$\tau(t) = \frac{f(t)}{2} U(t+\theta)\left|U(t+\theta)\right| \tag{29a}$$

θ represents the phase lag between $U(t)$ and the bottom shear stress $\tau(t)$ at the upper limit of the boundary layer.

Computed shear stresses for the more complex velocity case (irregular wave obtained by the non-linear propagation of an input sinusoidal wave) is presented in figure 9c. A comparison of results 1 and 3 with result 2 shows that result 1 is still closer to that of the K-L model than to result 3. Also, a slight discrepancy can be seen for the maximum value.

Despite the "*turbulence memory effects*", the proposed formulation fits closely with the boundary layer model results for the three cases analyzed. Comparisons were made, however, assuming that results given by the K-L model correctly represent the real conditions. Moreover, some discrepancies occur, especially for the maximum values. Naturally, other numerical results and also experimental data should be considered in future works.

V – Conclusions

A comprehensive literature review of the bottom friction formulae was undertaken. The use of a 1DV numerical turbulent closure model of the K-L type provides results to obtain a fit for the parameterization of the friction coefficient due to a sinusoidal wave for both rough and smooth turbulent regimes. Taking into account the pressure gradient over the rough bed proposed by Sleath [12], a close agreement between experimental data and a parametric formulation has been shown.

The same numerical boundary layer model has been utilized to extend the above studies to the smooth turbulent regime, allowing the development of a friction parameterization in the wave-current interaction, as proposed by Soulsby *et al.* [14] for the rough turbulent case.

Finally, a time-dependent bottom shear stress parameterization for irregular waves, taking into account the phase shift between mean flow velocity and the shear stress is proposed. The "*mem-*

ory effect", which we suggest, is not, however, taken into account in the parameterization. This procedure should therefore be improved in the future, taking this effect into account.

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Notations

The following symbols are used in this paper:

a, a_i (i=1,2,3,4)	Fitting coefficients
A	Wave excursion length
b, b_i (i=1,2,3,4)	Fitting coefficients
c_1, c_2	Fitting coefficients
D_{50}	Mean diameter
f_c	Current-alone friction coefficient
$f_{c(r)}$	Current-alone friction coefficient for rough flow
$f_{c(s)}$	Current-alone friction coefficient for smooth flow
f_{cw}	Friction coefficient with wave-current interaction
f_p	Pressure gradient contribution to the wave friction
$f(t)$	Instantaneous wave friction coefficient
f_w	Wave-alone friction coefficient
f_{wg}	Global friction coefficient due to waves
$f_{w(l)}$	Wave-alone friction coefficient for laminar flow
f_{wp}	Global friction coefficient due to waves and to the mean pressure gradient acting on the bed roughness
$f_{w(r)}$	Wave-alone friction coefficient for rough flow
$f_{w(s)}$	Wave-alone friction coefficient for smooth flow
f_1, f_2	Weight functions
h	Water depth
I	Fitting coefficient
J	Fitting coefficient
k	von Kármán constant (0.40)
K	Turbulent kinetic energy
K_N	Nikuradse roughness
L	Length scale of the turbulence
m, m_i (i=1,2,3,4)	Fitting coefficients
n, n_i (i=1,2,3,4)	Fitting coefficients
n_l	Fitting coefficient
p, p_i (i=1,2,3,4)	Fitting coefficients
P_c	Pressure due to the current
q, p_i (i=1,2,3,4)	Fitting coefficients
R_c	Reynolds number due to current alone
R_w	Reynolds number due to wave alone
R_0	Constant ($2.5*10^5$)
R_1, R_2	Lower and upper limits, respectively
T_p	Wave period
u	Horizontal component of the velocity in the boundary layer
u_c	Depth-average mean velocity of the current
$\hat{U}_{cw}$	Maximum bottom shear stress with wave-current interaction
U_{max}	Maximum value of the instantaneous velocity vector
U_{rms}	Root-mean-square of orbital velocities
$U(t)$	Instantaneous velocity vector
U_w	Horizontal component of the wave velocity, along the flow
$\hat{U}_w$	Orbital velocity amplitude of the wave at the upper limit of the boundary layer
U_1	Maximum wave velocity value
U_2	Minimum wave velocity value
v	Vertical component of the velocity in the boundary layer
V_w	Horizontal component of the wave velocity, normal to the flow
x	Horizontal coordinate
X_1	$= \tau_c / (\tau_c + \hat{\tau}_w)$
X_2	$= \tau_c / \lvert \vec{\tau}_c + \vec{\tau}_w \rvert$
y	Vertical coordinate
y_1	$= \tau_m / (\tau_c + \hat{\tau}_w)$
y_2	$= \tau_m / \lvert \vec{\tau}_c + \vec{\tau}_w \rvert$
Y	Normalized maximum shear stress for smooth bottom in the wave-current interaction (Y_1, Y_2)
Y_1	$= \tau_{\max} / (\tau_c + \hat{\tau}_w)$
Y_2	$= \tau_{\max} / \lvert \vec{\tau}_c + \vec{\tau}_w \rvert$
z_0	Bottom roughness length
α, β	Functions
ϕ	Angle between the wave and the current directions
ρ	Density of the fluid
θ	Phase lag between $U(t)$ and $\tau(t)$
υ	Kinematic viscosity of the fluid
υ_t	Turbulent viscosity
τ_c	Bottom shear stress due to current alone
τ_m	Mean (cycle-averaged) bottom shear stress with wave-current interaction
τ_{max}	Maximum bottom shear stress in the wave-current interaction
$\tau(t)$	Instantaneous bottom shear stress vector
τ_w	Amplitude of oscillatory bottom shear stress

	due to waves alone
$\hat{\tau}_w$	Bottom shear stress due to waves alone
$\hat{\tau}_{wp}$	Global bottom shear stress due to waves and to the mean pressure gradient acting on the bed roughness
$\hat{\tau}_p$	Bottom shear stress due to the mean pressure gradient
τ_1	Maximum bottom shear stress
τ_2	Minimum bottom shear stress

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