Vortex chamber diodes as throttle devices in pipe systems. Computation of transient flow

Diodes de vortex comme régulateur de flux dans des systèmes des conduits. Calculation des écoulment transitoires

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ABSTRACT
A theoretical solution for the computation of transient flows in a pipe system with a vortex chamber diode, based on an energy balance and including an algorithm for numerical application in water-hammer programs, is presented. Moreover, the experimental results of several case studies provide a dimensioning concept.

RÉSUMÉ
Une solution théorique est présentée pour la calculation des écoulements transitoires dans un système de conduits avec diodes de vortex, basée sur une balance d'énergies et incluant un algorithm pour l'application numérique dans des programmes pour le coup de bélier. De plus, les résultats expérimentaux pour plusieurs études de cas fournit un concept pour le dimensionnement.

1. Introduction

Vortex chamber diodes have proved effective for controling transient pipe flows. Due to an asymmetric throttle behaviour, these devices are especially useful for dampening oscillations in pipe systems [*Giesecke*, *Horlacher* 1989, *Haakh* 1993]. Since 1929, vortex chamber diodes have been constructed in the same very simple way, throttling being based only on hydrodynamic effects without any movable parts. This simple design makes vortex chamber diodes very reliable, but the problem of computing transient flow for dimensioning still remains.
This paper presents a simple method for computing transient throttling effects which proves advantageous compared with previous computation models based on weight functions to describe the transient flow. This method can be applied to describe boundary conditions in water-hammer programs for modelling transient pipe flow. The advantage is, in concrete terms, that this computation method needs less memory capacity in a computer and numerical simulation runs and has good convergence and stability.

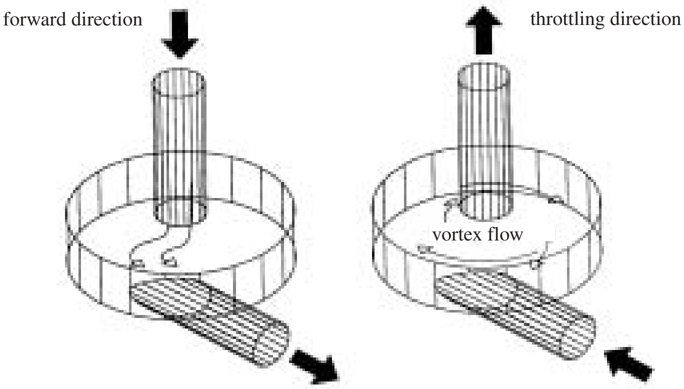

Fig. 1 Flow through a vortex diode: low throttling in forward direction, high pressure loss in throttling direction

2. Transient flow and time-dependent throttle losses of vortex chambers

Figure 2 shows the velocity v, the head loss h_v ($h_{v,VD}$) and ζ of a pipe system with a vortex diode under transient conditions ($t \leq 0$ s, $H_E = 0$, $t > 0$ s, H_E = const. $t > 0$) for forward direction (a) and throttling direction (b). For the forward direction, a vortex chamber produces a constant head loss coefficient ζ as each valve does, so that the transient flow can be described exactly by the Bernoulli equation

$$H_E = \frac{v^2(t)}{2g}(1 + {}'_{PL}) + \frac{dv(t)}{dt}\left(\frac{L_{Pl}}{g}\right) + h_{v,VD}(t) \qquad [mWS] \qquad (1)$$

with

$$h_{v,VD} = \frac{v^2(t)}{2\ g}\ (\zeta_{VD,forward}) \qquad (2)$$

This approach is not suitable for the throttling direction because here $\zeta_{VD,\,throttling}$ depends on time (see figure 2 b, where ζ reaches the stationary value after ≈ 7s). This is the main problem to be solved when computing transient flow.
If we add a vortex diode to a pipe system, the diode is like an additional energy store where the kinetic energy of the vortex flow (see chapter 3) determines the head loss of the vortex chamber. Experiments (stationary conditions) have shown that the kinetic energy E_{VD} in a vortex chamber diode is in proportion to the head

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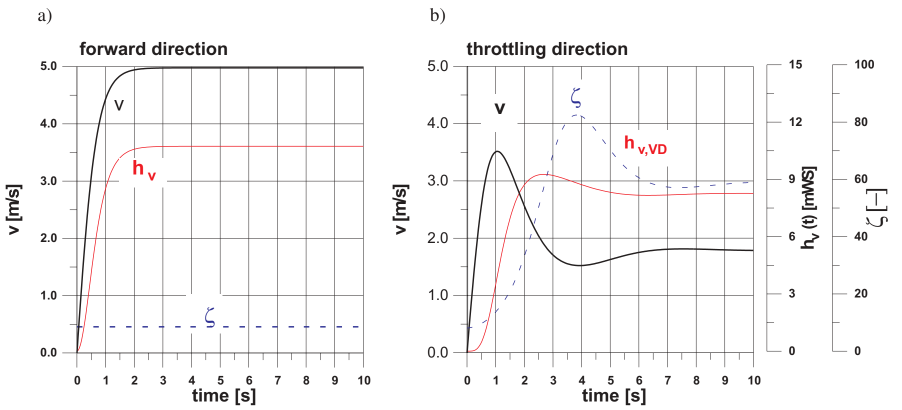

Fig. 2 Transient flow in a system of vortex chamber diode and pipe in forward direction (a) and in throttling direction (b) with v(t), $h_{vVD}(t)$ measured values of vortex diode, ζ computed

loss in the vortex diode. Factor K_{Ekin} [Nm/(mWS)] describes the relationship between the kinetic energy E_{VD} in the diode and the throttle losses $h_{v,VD}$. $h_{v,VD}$ can be computed by

$$h_{v,VD} = \frac{E_{VD}}{K_{Ekin}} \quad [mWS] \tag{3}$$

In the following, *equation (3)* will also be used to describe *transient* flow, assuming quasi steady-state conditions with

$$h_{v,VD}(t) = E_{VD}(t)\frac{1}{K_{Ekin}} \quad [\mathrm{m}] \tag{4}$$

For transient flow, this means that the throttle loss $h_{v,VD}$ (t) is also in proportion to the time- dependent kinetic energy E_{VD} (t) in the diode.
This leads to the 'extended' Bernoulli equation with

$$H_E = \frac{v^2(t)}{2g}(1+\zeta_{PL}) + \frac{dv(t)}{dt}\left(\frac{L_{Pl}}{g}\right) + \frac{E_{VD}(t)}{K_{Ekin}} \quad [mWS] \tag{5}$$

From eq. (3), E_{VD} can be calculated for stationary conditions, but how the kinetic energy within the vortex chamber will change during the transient period is still unknown. This will be derived in the following.
Under stationary conditions, the input (dE^+) of kinetic energy to the vortex can be calculated (with $E = \frac{1}{2} mv^2$) by $dE^+ = \frac{1}{2} \psi \cdot A \cdot v \cdot v^2$. The part of the kinetic energy of the vortex flow lost by dissipation dE^- is still unknown. To determine it, an approach in the form $dE^- = \xi \cdot h_{v,VD}$ will be used, so we have the loss of kinetic energy of the vortex flow as part of the total head loss of the diode. Stationary conditions mean a balance between energy input and dissipation, and this boundary condition yields

$$\frac{dE_{VD}}{dt} = \frac{1}{2}\,\rho A\; v\; v^2 - \eta \cdot h_{v,VD} \overset{!}{=} 0 \tag{6}$$

It is evident that η must depend on the typical geometric parameters and is specific to each vortex diode. Observing that $\rho Agv\, v^2/2g = \eta\, v^2/2g\, \zeta_{VD}$ and with $v = (2g\, h_{v,VD} / \zeta_{VD})^{1/2}$, we can determine η as

$$\xi = \frac{\psi \cdot A \cdot g}{\zeta_{VD}}\sqrt{\frac{2g}{1+\zeta_{VD}+\zeta_{PL}}}\sqrt{H_E} = \frac{\psi \cdot A \cdot g}{\zeta_{VD}}\sqrt{\frac{2g}{\zeta_{VD}}}\sqrt{h_{v,VD}} \quad \left[\frac{N}{s}\right] \tag{7}$$

We were thus able to derive η for stationary conditions. In accordance with the first assumption with $h_{v,VD}(t) = E_{VD}(t) / K_{Ekin}$, η from eq. 7 can be used for the computation of *transient* flow, so that we can calculate the unknown kinetic energy E_{VD} (t) of the 'extended' Bernoulli equation (5). We substitute $h_{v,VD}$ in eq (6) using eq. (4) and obtain the differential equation for E_{VD} in the form

$$\frac{dE_{VD}(t)}{dt} = \frac{1}{2}\,\psi A\; v\; v^2 - \xi\,\frac{1}{K_{E_{kin}}}\,E_{VD}(t) \tag{8}$$

Integration yields

$$E_{VD}(t) = \frac{1}{2}\,\psi A \cdot \int_0^t v(t)^3\, dt - \frac{\psi A g}{\zeta_{VD}}\sqrt{\frac{2g}{\zeta_{VD}}}\int_0^t h_{v,VD}^{\frac{3}{2}}(t)\,dt \quad [Nm] \tag{9}$$

We thus have the system of the two differential equations (5) ('extended' Bernoulli equation) and (9) which can be solved step by step for the two variables v(t) and $E_{VD}(t)$ for given boundary

conditions (for example $E_{VD}(t = 0) = 0$, $v\ (t = 0) \rightarrow$ figure 2b and figure 6).
The next step is to ascertain whether the assumptions correspond with reality. In numerous experiments, v(t) and $h_{v,VD}(t)$ were sampled. With v(t) and $h_{v,VD}(t)$ and ζ_{VD} (for stationary conditions), eq. 9 allows us to calculate $E_v(t)$. Figure 3 shows the correlations between $E_v(t)$ and $h_{v,VD}(t)$ ($r^2 = 0{,}99$) for *transient* conditions.

3. Determination of vortex diode characteristics

The most important form parameters for dimensioning a vortex diode are the diameter, the relation of diameter and height (d/h [-]) and the radii of the inlet and outlet jet (r_i, $r_o \approx ½$ pipe diameter). In dependence on the energy height, these data determine the vortex flow. Furthermore, we must take into account that a vortex flow is characterized by tangential velocity. According to *Haakh* (1994), the tangential velocity profile can be described as

$$v(r) = v_i(\frac{r_i}{r})^k(1-e^{-r^2\frac{R^2}{r_s^2}}) \qquad (10)$$

where k ([-]) describes the influence of friction on tangential velocity, r_s the vortex radius (condition: dv(r)/dr = 0; see *figure 4*) and R² defines the tangential velocity maximum. *Figure 4* shows tangential velocity profiles $v(r) / v_i(\frac{r_i}{r})^k$ versus r/r_s for different diodes compared with the Rankin vortex (no friction).
For dimensioning, a rule of thumb is 'inlet diameter ≈ outlet diameter ≈ vortex chamber height' (figure 4). What is decisive for dimensioning is the proportionality factor K_{Ekin} [Nm/m WS]. A standardized proportionality factor K^*_{Ekin} [1] [Nm/m WS/m³] was derived for a vortex diode with the known relation d/h (diameter/height) and the chamber height 1 m. We then obtain K_{Ekin} as

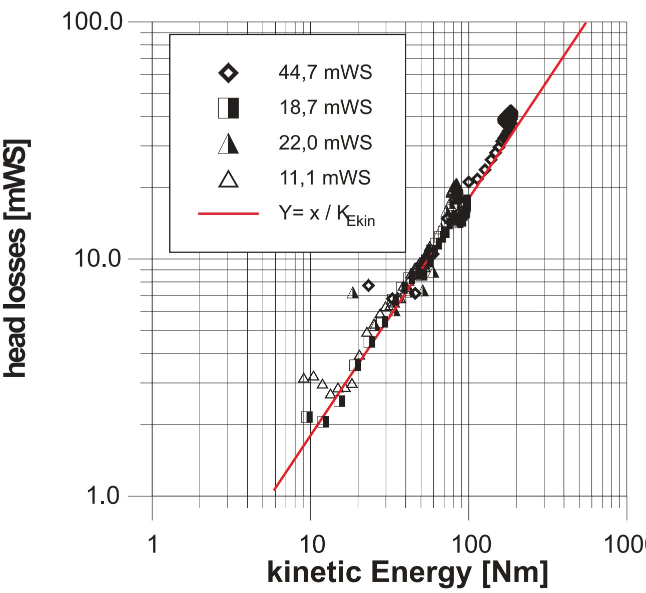

Fig. 3 Measured throttle loss h_v (t) [m WS] in dependence on kinetic energy E_{VD} (t) with H_E = 11,1, 18,7, 22,0 and 44,7 mWS

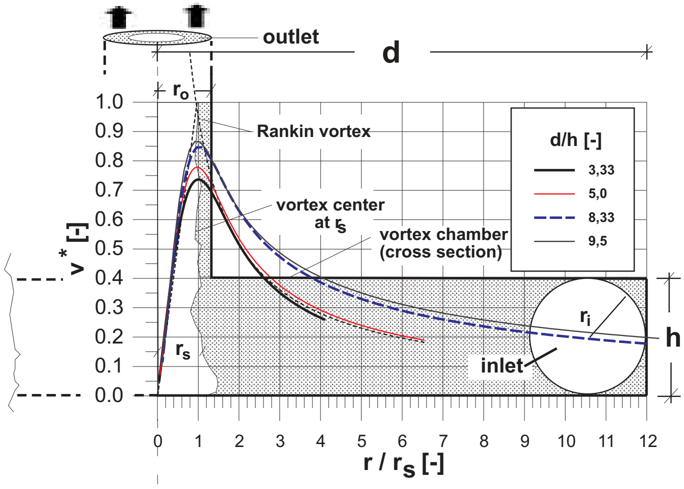

Fig. 4 Cross section of vortex diode and tangential velocity profile versus a standardized radius r/r_s [-]

$$K_{Ekin} \quad = \quad r_s^2 \cdot h \cdot K^*_{Ekin} \qquad (11)$$

In figure 5, we find a maximum of r_s/r_0 at d/h ≈ 10 (5a) and the influence of friction on the tangential velocity profile increases linearly with d/h (5b). R^2 and K^*_{Ekin} also increase with d/h (5c,f). An optimum for ζ_{VD} as well as for v_{max}/v_i can be estimated for d/h ≈ 9,5 (figure 5d, 5e).
Characteristics of vortex diodes are given in *Table 1*. These values are based on experiments and were gained by regression analysis and give the characteristics in a good approximation.

Table 1 Relations between vortex diode characteristics in dependence on d/h for 3 < d/h < 15, determined by regression analysis (based on investigations)

para-meter	**dimen-sion**	**Impact**	**regression formula**	**r²**
k	[-]	friction	1,149 - 0,052 d/h	0,996
v_i^*	[-]	v(r)-profile	0,4295 - 0,06041 d/h + 0,00351 (d/h)²	0,933
r_s/r_i	[-]	vortex radius	0,591 + 0,0328 d/h - 0,00155 (d/h)²	0,932
ζ_{WD}	[-]	loss coefficient	19,54 + 5,97 d/h - 0,308 (d/h)²	0,977
K^*_{Ekin}	[Nm/m WS/m³]	kinetic energy	-22415+ 18351 d/h + 1858 (d/h)²	0,999
v_{max}/v_i	[-]	efficiency	-0,315 + 1,107 d/h - 0,0591 (d/h)²	0,980
R²	[-]	v(r)-profile	1,0001 + 0,0425 d/h	0,995

[1] For example: If d/h = 5, d = 0.3 m, K^*_{Ekin} = 121500, r_e = 0,71625 then $K_{Ekin} = 121500 \cdot (0{,}71625 \cdot 0{,}003)^2 \cdot 0{,}06 = 3{,}366$ [Nm/m WS].

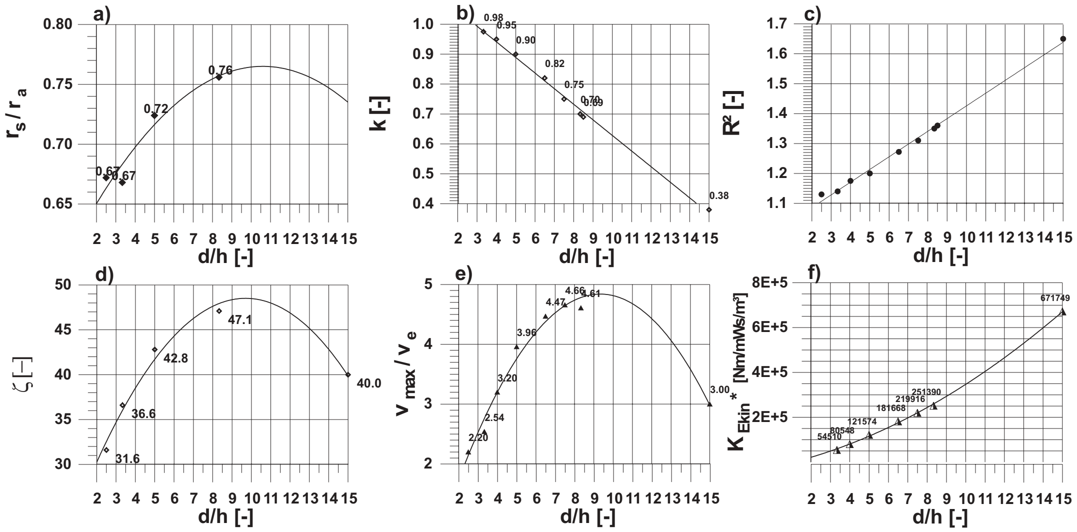

Fig. 5 Characteristics for dimensioning vortex diodes in dependence on d/h for 3 < d/h < 15, determined by regression analysis

4. Numerical simulation of transient flow

4.1 Assumption of incompressibility - computation by 'extended' Bernoulli equation

If $\frac{dv}{dt}$ respectively $\frac{dp}{dt}$ change slowly, transient flow can be computed by truncation and step-by-step integration of the Bernoulli equation (5) and the energy equation (9). We obtain

$$E_{VD}[i] = E_{VD}[i-1] + \underbrace{\frac{1}{2}\,\psi\cdot A\,(v[i-1]+\tfrac{1}{2}\Delta v[i-1])^3\cdot\Delta t}_{\Delta E_{VD}[i]} - \frac{\psi\cdot A\cdot g}{\zeta_{VD}}\sqrt{\frac{2g}{\zeta_{VD}}}\cdot h_{v,VD}[i-1]^{\frac{3}{2}}\cdot\Delta t \quad [Nm] \tag{12}$$

From *equation 5*, it follows that

$$\Delta v[i] = \begin{bmatrix} H_E[i]-h_{v,VD}[i]-(v[i-1] \\ +\frac{1}{2}\Delta v[i-1])^2/2g\cdot(1+\zeta_{PL}) \end{bmatrix}\frac{g}{L}\cdot\Delta t \\ \left\{+\Delta E_{VD}[i]\cdot\frac{1}{\psi\cdot g\cdot A}\,\frac{1}{\Delta v[i-1]}\right\}\frac{g}{L}\cdot\Delta t \quad \text{for } dE/dt<0 \tag{13}$$

Thus, the velocity can be determined

$$v\,[i] = v\,[i-1]+\Delta v\,[i] \tag{14}$$

This computation method produces quite good results if the time increment is sufficiently small. The criterion is $\Delta t \langle \ \frac{1}{10}\frac{L}{H_E\cdot g}$.

Figure 6 shows measured and computed results. The data given are based on 400 measurements at five different vortex chamber diodes. The computed values were gained by an 'extended' Bernoulli equation (eq 5), assuming incompressible conditions. The computation results were gained by solving equations 12, 13, 4 and 14 step by step, assuming an incompressible fluid. The experiments were carried out for one vortex diode (d = 0,5 m, h = 0,06 m, r_a = 0,025m = VD 2) at two energy heights and for two further vortex diodes (d = 0,9 m, h = 0,06 m, r_a =0,025 m = VD 4; d = 0,3 m, h = 0,06 m, r_a = 0,025 m = VD 1). *Figure 7* below shows the correlation between the measured and the computed head losses *(eq.4)*. Regression analysis *(figure 7)* yields a correlation between '$h_{v,VD}$ computed' and '$h_{v,VD}$ measured' with $r^2 > 0{,}99$.

4.2 Vortex diode as a boundary condition in water-hammer programs

In water-hammer computation programs, vortex diodes can be treated like other throttle devices. Most programs need a loss coefficient in dependence on time (ζ_{VD} (t)). In order to obtain this coefficient, either $h_{v,VD}$ and Q_p have to be determined from equations 12 and 13 for the nodal point of each time slot 'i' by a predictor-corrector-method and in an iterative solver or, alternatively, the loss coefficent is computed by approximation. Here, one starts each time from the computed value of the preceding time slot 'i - 1' and continues as follows

$$\zeta_{VD}[i] = \frac{2g(h_{v,VD}[i-1]+\frac{1}{2}\Delta h_{v,VD}[i-1])}{(Q[i-1]+\frac{1}{2}\Delta Q[i-1])^2}A^2 \tag{15}$$

Introducing the approach with $K = A^2\,\frac{g}{\zeta}$

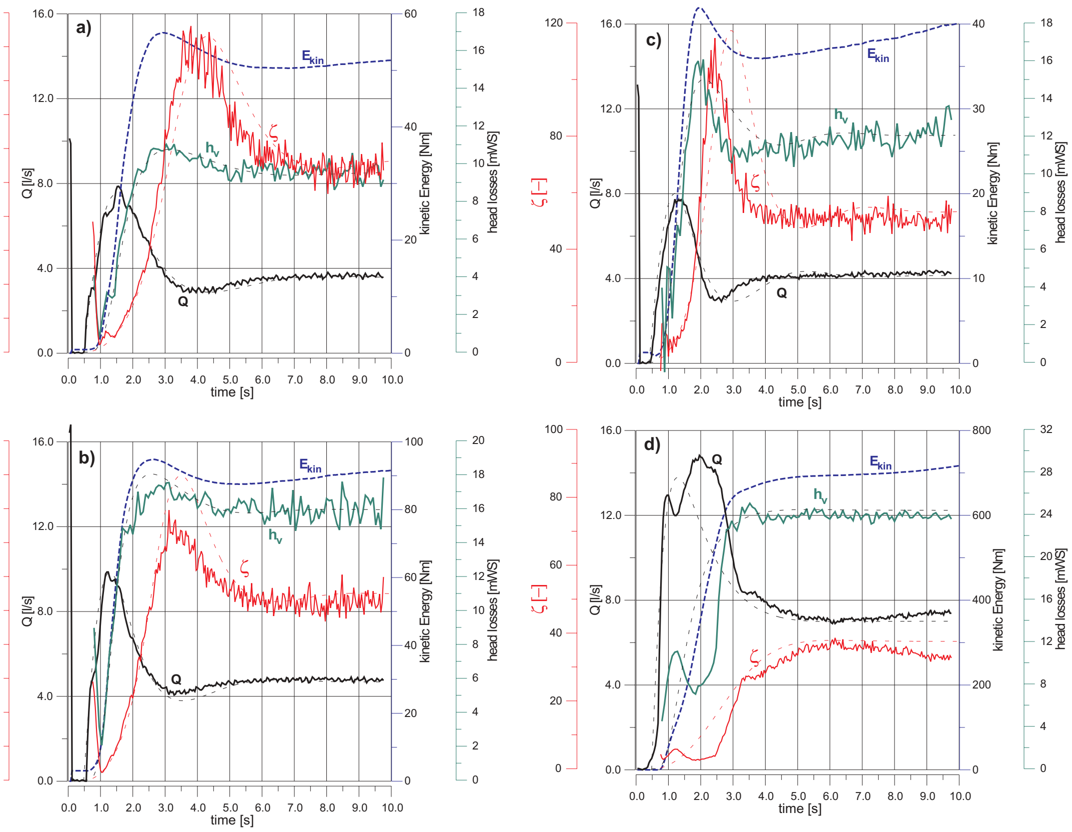

Fig. 6 Measured („---„ dashed lines) and computed values for step response of the system vortex diode/pipe:
a) VD2, H_E = 11,7 m WS b) VD 2, H_E = 18,1 m WS
c) VD 1, H_E = 14,1 m WS d) VD 4, H_E = 44,7 m WS

respectively $K_D = \frac{\vartheta_0 Q_0^2}{2h_{d0}}$ [*Horlacher, Lüdecke 1992*], we must taken into consideration that $\zeta_0 \approx \zeta_{v,VD}$ for the time before the vortex flow begins

($\vartheta = \sqrt{\zeta_{v,VD}} \,/\, \sqrt{\frac{2gh_{v,VD}}{Q_p^2/A^2} + \zeta_{v,VD}}$; no division by zero!).

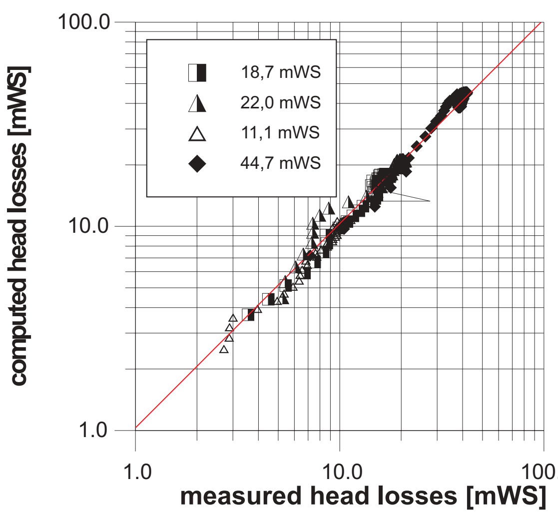

Fig. 7 Measured and computed head losses (synopsis of four measurements with H_E = 11,1, 18,7, 22,0 and 44,7 mWS)

It should also be taken into account that throttling works only in one direction (Index 'v'). This means that the quadratic equation for introducing the compatibility condition has different results depending on the flow direction

($K_1 = A^2 \frac{g}{\zeta_{v,VD}}$; $K_2 = A^2 \frac{g}{\zeta_{f,VD}(t)}$).

4.3 Dissipation of vortex flow

A further state of transient flow is the process of decay when the energy supply is reduced or stopped and the vortex flow dissolves under the influence of friction ($\eta \bullet h_{v,VD}$ (t)). *Equation 5* cannot be applied in this case for dE/dt < 0 (*figure 8*). Here, it must be taken into account that, because of the high energy of the vortex flow, the loss coefficient ζ (t) is rather high in relation to the decreasing velocity (in the pipe system) (v_i).
The reason is that the kinetic energy of the vortex flow diminishes more slowly during the process of decreasing flow (dE/dt < 0) than the flow in pipe system decreases. This means dE/dt < dv/dt. For this reason, the throttling vortex still exists when Q = 0 (inertia reaction of rotating mass). More detailed investigations into dissolving vortex flows in turbulent conditions are being made at present [*Giesecke, Horlacher, Raichle* 1996]. *Figure 8* shows that H_E (t) decreases linearly (within 10 seconds to zero), E_{VD} (t_{END} = 30 s) ≈ 20 Nm and $h_{f,VD}$ = 4 m WS.

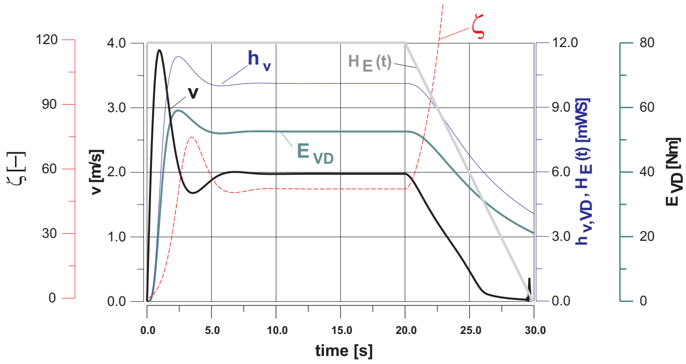

Fig. 8 Hydraulic characteristics profile for transient flow phases (pipe: L = 15,63 m, r = 0,025m;vortex diode: K_{Ekin} = 5,2; h = 0,05m;

5. Example - surge chamber oscillation at *Kaunertalkraftwerk*

In the Kaunertalkraftwerk of TiWAG (*Seeber* 1970)**,** there is a pressure tunnel of 13,178 km length (diameter 4 m) with throttled surge chamber by a vortex diode, installed to control back currents and pressure oscillations, caused by unsteady flows when the hydropower station is in operation. The dimensions of the vortex diode are d = 6,4 m, h = 1,64 m, A_D = 2,11 m², d/h = 3,9 [-]. In accordance with figure 5 (respectively with diagram 5f)**,** the value of K^*_{Ekin} = 77414. The vortex radius is 1/2 • 1,64 • (0,695) = 0,57 m. Then, K_{Ekin} = 41,25 kNm/m WS.
The experimental data published by Seeger [*Seeger* 1970] were computed on the basis of the parameters given above (*figure 9*). It is possible to describe the pressure development before inlet and downstream of the throttle device as well as $h_{v,\,max}$ and the remaining oscillation. There is, however, a phase shift between the numerical model and the experimental results, because the throttle-loss profile shows a flatter shape in the numerical simulation, although Δh and $h_{f,\,max}$ are decisive for the design.

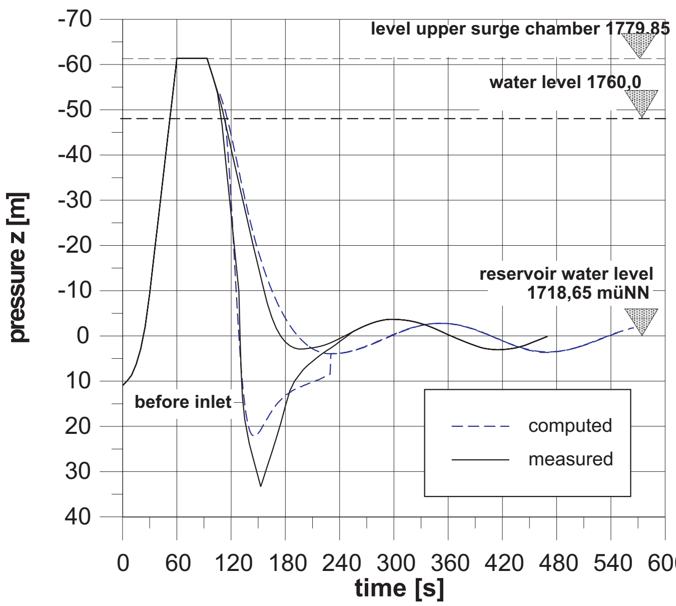

Fig. 9 Profile of water chamber oscillation at Kaunertalkraftwerk (test on October 8, 1964, switching off 223 MW, experimental data by *Seeger* [1970], computed data according to equations 12-14)

6. Discussion of results and summary

The method introduced here enables the computation of transient flow in vortex diodes by investigating the kinetic aspects. Compared with methods that have been used up to now and that are based on weight functions as characteristics of every system, the process described here is distinctly easier, a fact that is especially important in numerical simulation.
It could be shown that the same proportional relation between throttle loss and kinetic energy describing states of steady flows is also valid for the whole process of transient flow. By a simple balance of kinetic energy, it is possible to determine throttle loss as a function of time as well as the loss coefficient ζ. Specific kinetic energy is the most important parameter for designing a vortex diode. Specific kinetic energy is in proportion to throttle loss, depends on dimensions and determines the hydraulic parameters as well as the question of how long it takes until oscillation has built up. These critical parameters are dependent on the dimensionless geometric relation d/h.
For the numerical simulation, the solution presented here may be used with the 'extended' Bernoulli equation, which is suitable for slow alterations of state, for example oscillations in water chambers or in systems with an air chamber and gas cushion, or as a computation module for water-hammer programs which provides the loss coefficient ζ as a function of time.
For processes of vortex-flow dissolution, this computation method provides acceptable results. Nevertheless, further investigations are necessary. The Institut für Wasserbau, Lehrstuhl für Wasserbau und Wasserwirtschaft, at the University of Stuttgart is at present engaged in research in this field. The examples presented here effectively prove the method for computing transient flow in pipe systems.

Notations

A	cross section area of pipe	[m²]
d	diameter	[m]
dE	infinitesimal change of kinetic energy	[Nm]
dE^+	infinitesimal increase of kinetic energy	[Nm]
dE^-	infinitesimal decrease of kinetic energy	[Nm]
dt	infinitesimal time slot	[s]
E	kinetic energy	[Nm]
g	acceleration by gravity	[m/s²]
h	height	[m]
h_v	pressure loss	[mWS]
H_E	energy height	[mWS]
K	numerical parameter	[m³/s²]
L	length	[m]
Q	flow rate	[m³/s]
r	radius	[m]
v	velocity	[m/s]
Γ	circulation	[m²/s]
ζ	loss coefficient	[-]
η	head loss of kinetic energy by dissipation	[N/s]
ρ	density	[kg/m³]
τ	integration period	[s]

ω	angular velocity	[m²/s]
Δ	finite quantity increase	

Indices

Ekin	kinetic energy
END	end
f	forward direction
i	inlet
max	maximum value
o	outlet
PL	pipe line
s	vortex core
*	standardized quantity
v	throttlind flow-direction
VD	vortex diode
0	zero; initial state

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